Tamalpais Union High School District
Larkspur, California

<u>Course of Study</u>
Advanced Placement Calculus AB / BC

I. INTRODUCTION

The advanced placement courses in calculus (both AB and BC) consist of a full academic year of work in calculus and related topics comparable to courses in colleges and universities. According to the State of California's Mathematical Standards for calculus, "when taught in high school, calculus should be presented with the same level of depth and rigor as are entry level college and university calculus courses. … Calculus is a widely applied area of mathematics and involves a beautiful intrinsic theory. Students mastering this content will be exposed to both aspects of the subject." The advanced placement calculus courses are intended for students who have thorough knowledge of college preparatory mathematics including algebra, axiomatic geometry, trigonometry, and analytic geometry (rectangular and polar coordinates, equations and graphs, lines, and conic sections).

Both calculus AB and BC are concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. Calculus BC is an extension of calculus AB rather than an enhancement; common topics require a similar depth of understanding. Both courses are challenging and demanding. District schools may offer either (or both) of these courses.

The district's curriculum for advanced placement calculus AB and BC will follow the College Board's AP calculus AB / BC Course Description. Teachers should annually monitor the College Board's website for the latest curricular changes, additions, or deletions.

This course addresses the following Tam 21st Century Mission, Beliefs, and Goals in the following ways:
The learning environment provides opportunities for students to
- Acquire, manage, and use knowledge of calculus concepts and skills (mission)
- Think critically and creatively when developing a conceptual understanding of calculus (mission)
- Practice self-directed learning, decision making, and problem solving (mission)
- Participate in a program for high achieving students (philosophy)
- Take part in the full learning experience: challenge, exploration, risk-taking, initiative, hard work, success and failure, and joy and enthusiasm (belief)

This course contributes to attainment of the following district outcomes:

Outcome 1 Communicate articulately, effectively, and persuasively when speaking and writing.

Outcome 2 Students will read and analyze material in a variety of disciplines

Outcome 3 Use technology as a tool to access information, analyze and solve problems and communicate ideas.

Outcome 5 Apply mathematical knowledge and skills to analyze and solve problems.

II. STUDENT LEARNING OUTCOMES

A. The students will

- earn a qualifying score of 3 or better on the AP Calculus exam;
- become active learners by investigating, conjecturing, verifying, applying, evaluating, and communicating mathematical ideas both collaboratively and individually in solving problems;
- explain and justify their work and thinking on "free response" problems that give them the opportunity to synthesize and apply a variety of concepts;
- apply the concepts associated with differentiation and integration to real world problems;and
- use appropriate technology (graphing calculators are required) as an integral part of student work.

In support of the California State Content Standards, the mathematics framework, and the outline of topics provided by the College Board, the AB topics may be organized into three topic families and the BC topics into four. For the AB syllabus, the topic families are a) Functions, Graphs, and Limits; b) Derivatives; c) Integrals. The BC syllabus also has the topic family of Polynomial Approximation and Series. The topic outline for calculus BC includes all calculus AB topics. Topics which are part of calculus BC and not part of calculus AB are found in paragraphs that are marked with a plus sign (+) or an asterisk (*). The additional topics can be taught anywhere in the course that the instructor wishes. Some topics will naturally fit immediately after their calculus AB counterparts. Other topics may fit best after the completion of the calculus AB topical outline. (See the Teacher's Guide-AP Calculus for sample syllabi.) Although the examination is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

Upon completion of the course, students will be qualified to take the advanced placement calculus exam for their respective course. The students will become accomplished in the following areas:

(*Note: These student learning outcomes are referenced to the State Content Standards for calculus. Number in parentheses indicates the Standard.)* Not all of the topics listed by the College Board as being essential for an advanced placement program are listed as standards by the State of California.

I. **Functions, graphs, and Limits**

Analysis of graphs. (9.0)

- The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits). (1.0)
- An intuitive understanding of the limiting process. (1.0)
- Calculating limits using algebra. (1.1)
- Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior.
- Understanding asymptotes in terms of graphical behavior.
- Describing asymptotes behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change. (For example, contrasting exponential growth, polynomial growth, and logarithmic growth.)

Continuity as a property of functions.
- An intuitive understanding of continuity. (Close values of the domain lead to close values of the range.) (2.0)
- Understanding continuity in terms of limits. (2.0)
- Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem.) (3.0)

***Parametric, polar, and vector functions.** The analysis of planar curves includes those given in parametric form, polar form, and vector form.

II **Derivatives**

Concept of the derivative.
- Derivative presented graphically, numerically, and analytically. (2.0)
- Derivative interpreted as an instantaneous rate of change. (4.2)
- Derivative defined as the limit of the difference quotient. (4.0)
- Relationship between differentiability and continuity. (4.3)

Derivative at a point.
- Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- Tangent line to a curve at a point and local linear approximation.
- Instantaneous rate of change as the limit of average rate of change.
- Approximate rate of change from graphs and tables of values.

Derivative as a function.
- Corresponding characteristics of graphs of f and f'. (9.0)
- Relationship between the increasing and decreasing behavior of f and the sign of f'.
- The Mean Value Theorem and its geometric consequences. (8.0)
- Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second Derivatives. (7.0)
- Corresponding characteristics of the graphs of f, f', and f''.
- Relationship between the concavity of f and the sign of f''.
- Points of inflection as places where concavity changes.

Applications of derivatives.

- Analysis of curves, including the notions of monotonicity and concavity. (9.0)
- \+ Analysis of planar curves given in parametric form, polar form, and vector form including velocity and acceleration vectors. (6.0)
- Optimization, both absolute (global) and relative (local) extrema. (11.0)
- Modeling rates of change, including related rates problems. (12.0)
- Use of implicit differentiation to find the derivative of an inverse function.
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed and acceleration. (4.2)
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations. (27.0)
- \+ Numerical solution of differential equations using Euler's method.
- \+ L'Hôpital's Rule, including its use in determining limits and convergence of improper integrals and series. (8.0)

Computation of derivatives.

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions. (4.4)
- Basic rules for the derivative of sums, products, and quotients of functions.
- Chain rule and implicit differentiation. (5.0)
- \+ Derivatives of parametric, polar, and vector functions. (6.0)

III. **Integrals**

Interpretations and properties of definite integrals.

- Computation of Riemann sums using left, right, and midpoint evaluation points. (13.0)
- Definite integral as a limit of Riemann sums over equal subdivisions. (13.0)
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: (15.0)

$$\int_a^b f'(x)dx = f(b) - f(a)$$

- Basic properties of definite integrals. (Examples include additivity and linearity.)

Applications of integrals.

- Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and the length of a curve (including a curve given in parametric form). (14.0, 16.0)

Fundamental Theorem of Calculus. (15.0)
- Use of the Fundamental Theorem to evaluate definite integrals.
- Use of the fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined. (15.0)

Techniques of antidifferentiation.
- Antiderivatives following directly from derivatives of basic functions. Antiderivatives by substitution of variables (including change of limits for definite integrals), parts (BC), and simple partial fractions (nonrepeating linear factors only) (BC). (17.0, 19.0)

\+ Improper integrals (as limits of definite integrals). (22.0)

Applications of antidifferentiation.
- Finding specific antiderivatives using initial conditions, including applications to motion along a line.
- Solving separable differential equations and using them in modeling. In particular, studying the equation y'=ky and exponential growth. (27.0)
    + Solving logistic differential equations and using them in modeling.

Numerical approximations to definite integrals.
- Use of Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

***IV. Polynomial Approximations and Series**

***Concept of series.**
A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence or divergence. (23.0)

***Series of constants.**
+ Motivating examples, including decimal expansion.
+ Geometric series with applications.
+ The harmonic series.
+ Alternating series with error bound.
+ Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of *p*-series.
+ The ratio test for convergence and divergence. (24.0)
+ Comparing series to test for convergence or divergence.

***Taylor series.** (26.0)
+ Taylor polynomial approximation with graphical demonstration of convergence. (For example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve.)
+ Maclaurin series and the general Taylor series centered at $x = a$.
+ Maclaurin series for the functions e , sin x, cos x and $\frac{1}{1-x}$
+ Formal manipulation of Taylor series and shortcut to computing Taylor series, including

substitution, differentiation, antidifferentiation, and the formation of new series from known series. (25.0)

- + Functions defined by power series.
- + Radius and interval of convergence of power series. (24.0)
- + LaGrange error bound for Taylor polynomials.

III. ASSESSMENT

A. <u>Student Assessment</u>

Ultimately, the students will be assessed by their scores on the advanced placement exam. Over the course of the year, students will be assessed in a variety of ways. They will have ample opportunity to practice the "free response" type of problem, multiple choice questions (including those with "none of these" as a choice), and short answer problems. It is suggested that students have the opportunity to synthesize their knowledge on practice and released advanced placement exams.

Assessments will be made on the basis of a variety of means such as quizzes, tests, projects, performance tasks, and investigations. Informal assessments may involve writing samples and daily work. Students will be given grading criteria and course expectations, preferably in writing at the beginning of the course.

B. <u>Course Assessment</u>

The success of the course will be judged by viewing longitudinal data that compares (1) the number of students who pass the advanced placement exam with a score of 3 or higher; (2)mean GPA of the course, and (3) the number of students who are able to place into the subsequent course in college.

IV. METHODS / MATERIALS

A. <u>Student Learning Activities</u>

Instruction will involve lecture, small group and individual investigation, and individual practice. Tasks should utilize sound and significant mathematics, engage students, develop student mathematical understandings and skills, stimulate students to reason and make connections, and promote communication about mathematics. Teachers will monitor student participation in discussions and group work, pose questions that challenge thinking, and ask students to clarify and justify their ideas orally and in writing.

It is suggested that students have the opportunity to synthesize their knowledge on practice and released advanced placement exams.

B. <u>Materials/ Technology</u>

This course uses the Board-adopted textbook <u>Calculus: Early Transcendentals Single and Multivariable</u> (8^{th} edition) by Anton, Bivens, Davis

Graphing calculators are a required element of this course. Half of the advanced placement exam, both AB and BC, require the use of the graphing calculator. Resources such as the College Board website, Paul Forester's Calculus, Lin McMullan's Advanced Placement Calculus Practice Exams, and the current Cliff's Notes for AB / BC Calculus should be utilized in conjunction with the textbook and resource materials.

C. School to Career Goals

Whenever appropriate, the instructor will demonstrate how the concepts learned are applied in the professional world. Calculus is application oriented so there will be ample opportunity to incorporate "real world" applications. In addition, guest speakers may come to the class to give their personal accounts of the utility of calculus and its applications.

D. Suggested Instructional Time Allocation

See the "Teacher's Guide-AP Calculus" for sample syllabi at apcentral.com under the calculus heading.

V. GENERAL INFORMATION

A. Prerequisites

The prerequisite for calculus AB is a C or better in precalculus 1 – 2. The prerequisite for calculus BC is a B or better in precalculus 1 – 2. It is recommended that students earning a B in precalculus meet with their current precalculus teacher before deciding which course, calculus AB or calculus BC, would be most appropriate for them. Most B-in-precalculus students would be appropriately placed in the AB course rather than the BC course.

B. Requirements Met

Successful completion of AP calculus AB or BC is accepted as partial fulfillment of the UC/CSU "C" admissions requirement. The course satisfies 10 credits of the Tamalpais Union High School District's 30 credit mathematics graduation requirement.

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