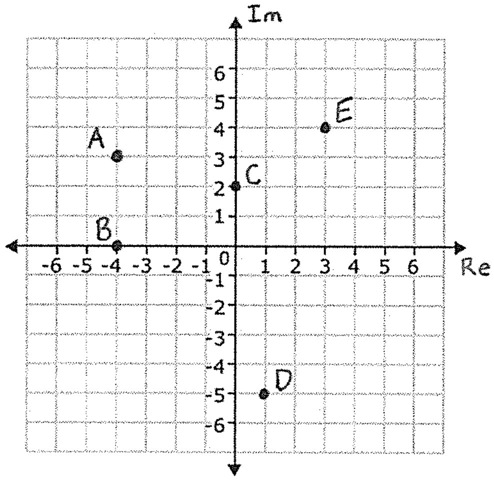
Practice Test

Complex Numbers

To receive full credit, all complex number answers must be written in the form $a + bi$, with $a, b \in \mathbb{R}$.

1. Identify each number indicated on the complex plane below.



A: $-4+3i$

B: -4

C: $2i$

D: $1-5i$

E: $3+4i$

Find the absolute value of each complex number.

2. $|1 + 6i| = \sqrt{1^2+6^2}$

$\boxed{=\sqrt{37}}$

3. $|27i| = \sqrt{27^2+0^2}$

$\boxed{=27}$

Perform the indicated operations.

4. $(5 + 7i) - (4 - i)$

$\boxed{1+8i}$

5. $i(6 + 2i)$

$6i+2i^2$

$\boxed{-2+6i}$

6. $6 + (2 - 8i)$

$\boxed{8-8i}$

7. $\frac{4-3i}{i} \cdot \frac{i}{i} = \frac{4i-3i^2}{-1}$

$\boxed{-3-4i}$

8. $(1 + i)(1 + 2i)$

$1+2i+i+2i^2$

$\boxed{-1+3i}$

9. $(3 - 5i) - (3 + 5i)$

$\boxed{-10i}$

10. $(4 - bi)(4 + bi)$

$16-4bi+4bi-b^2i^2$

$\boxed{16+b^2}$

11. $\frac{3+i}{1+i} \cdot \frac{1-i}{1-i} = \frac{3+i-3i-i^2}{2}$

$= \frac{4-2i}{2} = \boxed{2-i}$

12. Suppose $f(x)$ is a quartic polynomial that has at least two complex roots: $-2i$ and $3+5i$. Will the graph of $f(x)$ cross the x-axis? Explain.

A quartic polynomial (degree 4) can only have 4 factors thus only four roots. Since complex conjugates are also roots, we have 4 complex roots $(-2i, 2i, 3+5i, + 3-5i)$ so there is no room left for a real root. $\boxed{\text{No it will not.}}$

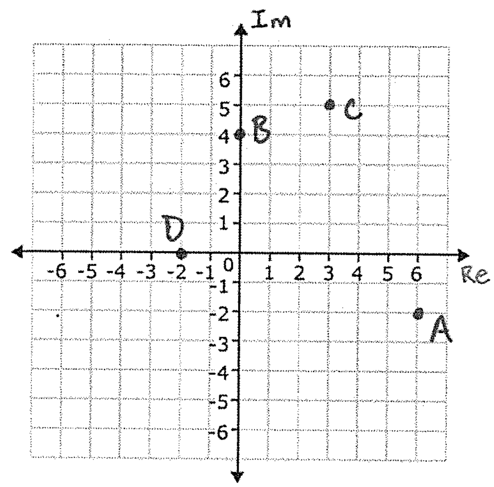
13. Graph and label the four points below on the complex plane.

A: $6-2i$

B: $4i$

C: $3+5i$

D: -2



Simplify each expression.

14. $i^{18} = i^{16}\cdot i^{2} = 1\cdot i^{2} = \boxed{-1}$

15. $-i^{7} = -i^{4}\cdot i^{3} = -1\cdot -i = \boxed{i}$

16. $i^{8,000,001} = i^{8000000}\cdot i = 1\cdot i = \boxed{i}$

17. $\sqrt{7i^{24}-2i^{2}} = \sqrt{7\cdot 1-2(-1)} = \sqrt{9} = \boxed{3}$

18. Which of the following numbers is its own complex conjugate? Explain.

$3i$ $\qquad$ 7 $\qquad$ $1+i$

$7 = 7+0i$ so its complex conjugate is $7-0i = 7$.

19. Is the following statement true or false? Explain or provide an example to show you are correct.

If $f(x)$is a polynomial with real coefficients and $f(2)=0$, then we must also have $f(-2)=0$.

$\boxed{\text{False}}$ Consider the polynomial $f(x) = x-2$
This polynomial has real coefficients with $f(2)=0$ but $f(-2)\neq 0$.

Find polynomials with real coefficients that have the following roots.

20. $x = 6i$, $-6i$

$(x-6i)(x+6i)$

$x^2-6ix+6ix-36i^2$

$\boxed{x^2+36}$

21. $x = 5+i$, $5-i$

$(x-(5+i))(x-(5-i))$

$x^2-(5+i)x-(5-i)x+(5+i)(5-i)$

$\boxed{x^2-10x+26}$

22. $x = 3$

$\boxed{x-3}$

23. $x = 1$ and $x = 2i$, $-2i$

$(x-1)(x-2i)(x+2i)$

$(x-1)(x^2+4)$

$\boxed{x^3-x^2+4x-4}$

Find all roots, both real and complex, of the following functions.

24. $f(x) = x^3 + 2x^2 + 2x$

$0 = x(x^2+2x+2)$

$x=0 \qquad x^2+2x+2=0$

$x = \frac{-2 \pm \sqrt{2^2-4(2)(1)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{-4}}{2}$

$\boxed{x=0, -1 \pm i}$

25. $f(x) = 2x^4 - 2$

$0 = 2(x^4-1)$

$0 = 2(x^2+1)(x^2-1)$

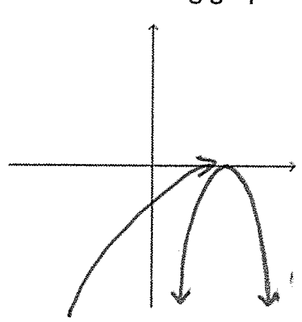
$0 = 2(x^2+1)(x+1)(x-1)$

$x^2+1=0 \qquad x+1=0 \qquad x-1=0$

$\boxed{x = \pm i, \pm 1}$

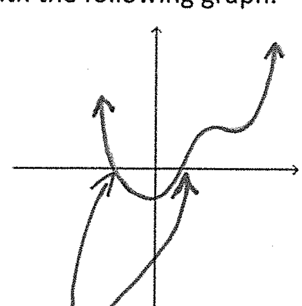
Based on its description and graph, determine how many real and complex roots each function will have.

26. $f(x)$ is a quadratic function with the following graph.



1 real root (w/multiplicity 2)
0 complex roots

27. $f(x)$ is a quartic function with the following graph.



2 real roots
2 complex roots