Practice Test 4
Inverse Functions

Use the three tables of values below to evaluate questions 1–6.

x	$f(x)$
−2	4
2	3
5	−6
0	−3

x	$g(x)$
3	5
4	5
−3	0
5	2

x	$h(x)$
−3	0
−4	0
−5	4
−6	−4

1. $(f \circ g)(3)$

$= f(5) = \boxed{-6}$

2. $h(f(0))$

$= h(-3) = \boxed{0}$

3. $(h \circ h)(-6)$

$= h(-4) = \boxed{0}$

4. $g(h(-5))$

$= g(4) = \boxed{5}$

5. $h\left(f(g(4))\right)$

$= h(f(5))$
$= h(-6) = \boxed{-4}$

6. $(g \circ f \circ g)(5)$

$= g(f(2))$
$= g(3) = \boxed{5}$

For questions 7–10, let $f(x) = x^2 - 5$, $g(x) = 2x + 3$, and $h(x) = x$. Find an expression for each composition.

7. $g(f(x)) = g(x^2-5)$

$= 2(x^2-5) + 3$

$\boxed{g(f(x)) = 2x^2 - 7}$

8. $(f \circ h \circ h)(x) = f(h(x))$

$= f(x) = x^2 - 5$

$\boxed{(f \circ h \circ h)(x) = x^2 - 5}$

9. $(f \circ g)(x) = f(2x+3)$

$= (2x+3)^2 - 5$

$\boxed{(f \circ g)(x) = 4x^2 + 12x + 4}$

10. $g\left(h(g(x))\right) = g(h(2x+3))$

$= g(2x+3) = 2(2x+3) + 3$

$\boxed{g(h(g(x))) = 4x + 9}$

11. Suppose f is a function such that $f = f^{-1}$. What must $f\left(f(3x^7)\right)$ equal?

$\boxed{f(f(3x^7)) = 3x^7}$ (f is its own inverse... it undoes itself. Since $f = f^{-1}$, $f \circ f = f \circ f^{-1} = Id$)

12. Using the words "input" and "output," explain what the identity function is.

The identity is the function whose output always equals its input.

Find the inverse of each function.

13. $f(x) = -4x$

$x = -4y$

$y = \frac{x}{-4}$

$\boxed{f^{-1}(x) = -\frac{x}{4}}$

14. $f(x) = 3x - 2$

$x = 3y - 2$

$3y = x + 2 \qquad y = \frac{x+2}{3}$

$\boxed{f^{-1}(x) = \frac{x+2}{3}}$

15. $f(x) = (x-7)^5$

$\sqrt[5]{x} = \sqrt[5]{(y-7)^5}$

$y - 7 = \sqrt[5]{x} \qquad y = \sqrt[5]{x} + 7$

$\boxed{f^{-1}(x) = \sqrt[5]{x} + 7}$

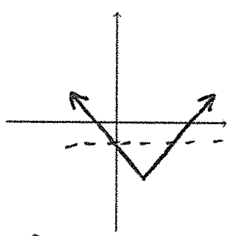
16. $f(x) = \frac{\sqrt[3]{x}+4}{5}$

$x = \frac{\sqrt[3]{y}+4}{5} \rightarrow 5x = \sqrt[3]{y} + 4$

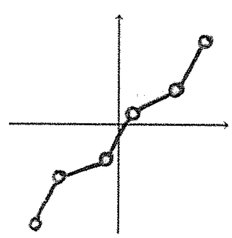
$\sqrt[3]{y} = 5x - 4 \rightarrow y = (5x-4)^3$

$\boxed{f^{-1}(x) = (5x-4)^3}$

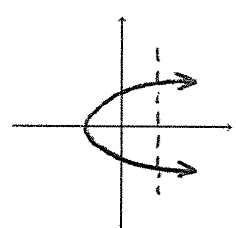
17. Identify each of the following relations as either "not a function," "function, not one-to-one," or "one-to-one function."



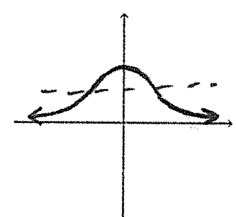
function, not 1-1



1-1 function



not a function

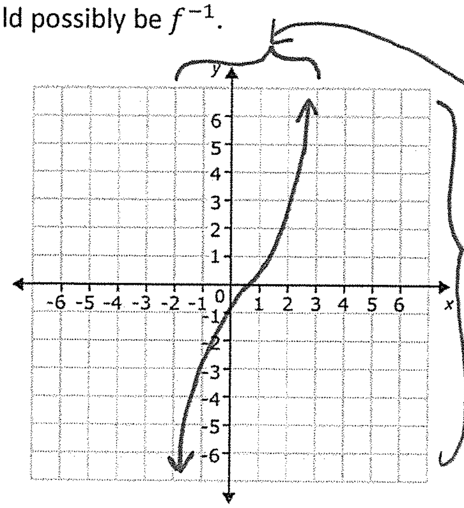


function, not 1-1

18. a) The domain and range of a function, f, are D: $(-\infty, \infty)$ and R: $-2 < y < 3$. Assuming f is one-to-one, what must the domain and range of f^{-1} be?

D: $(-2, 3)$ R: $(-\infty, \infty)$

b) Graph a function that could possibly be f^{-1}.



Checking for 3 things:

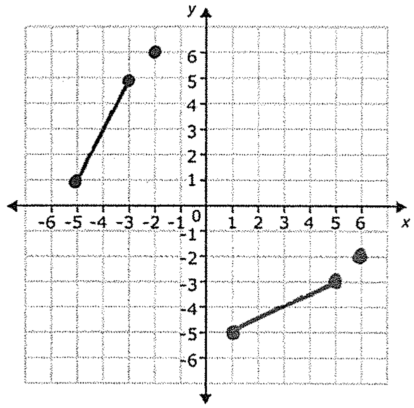
① Is your domain right?

② Is your range right?

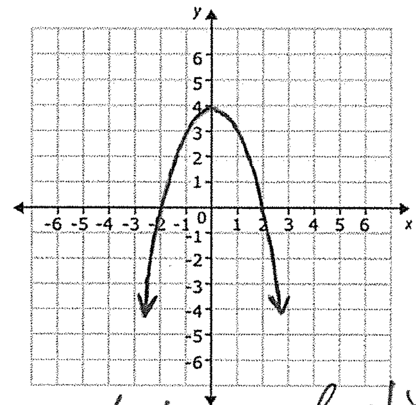
③ Did you draw a 1-1 function?

If the functions below have an inverse function, graph it on the same plane as the original function. If an inverse function does not exist, write "no inverse function."

19.

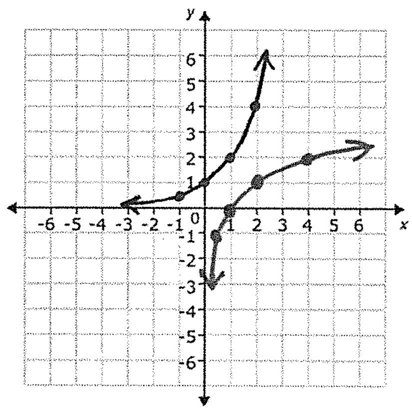


20.

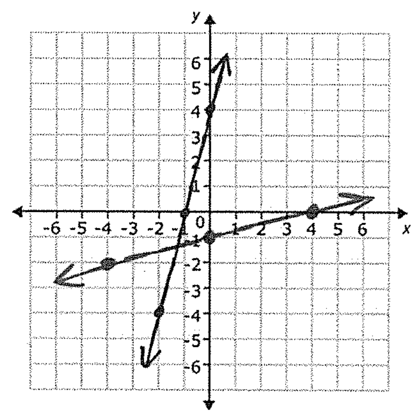


No inverse function

21.



22.



Label each of the following statements as "true" or "false." If a statement is true, explain why it is true. If a statement is false, provide an example that demonstrates that it is false.

23. The only function that is its own inverse is the identity function.

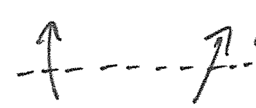
False. Just this little table proves it wrong.

x	$f(x)$
2	1
1	2

Or, you can list examples like $f(x)=-x$

24. An even degree polynomial function is never one-to-one.

True. Since end behavior matches on either side we will always have outputs w/ multiple inputs.



← fails HLT.

25. If $(3,-5)$ is a point on the one-to-one function f, then $(-3,5)$ must be a point on the function f^{-1}.

False. The only point that must be on f^{-1} is $(-5,3)$. $(-3,5)$ doesn't matter.

26. $(f \circ f^{-1} \circ f)(x) = f(x)$

True. This is just the identity, so it can be replaced by x.