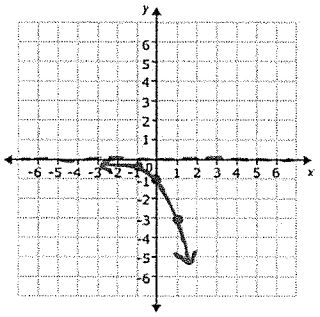
Practice Test 5

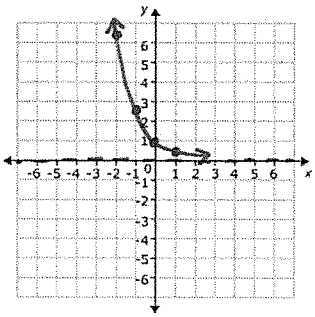
Exponential Functions

Graph the following functions. You must plot at least three clear points and any asymptotes with a dashed line.

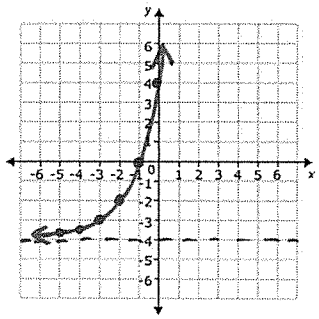
1. $f(x) = -3^x$



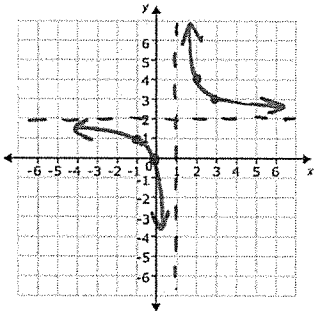
2. $f(x) = \left(\frac{2}{5}\right)^x$



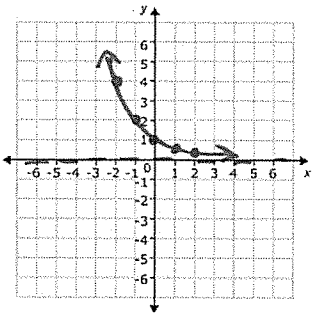
3. $f(x) = 2^{x+3} - 4$ ← left 3, ← down 4



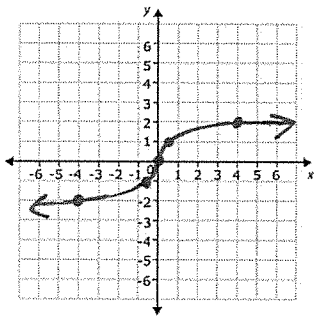
4. $f(x) = \frac{2x}{x-1}$



5. $f(x) = 2^{-x} = \left(\frac{1}{2}\right)^x$



6. $f(x) = \sqrt[3]{2x}$



x	y
0	0
$\frac{1}{2}$	1
$-\frac{1}{2}$	-1
4	2
-4	-2

Suppose you invest $3,000 in the savings accounts described in questions 7–10. How much money would be in each account after 5 years?

7. Bank of America pays 3.5% compounded semiannually.

$$A = 3000\left(1 + \frac{0.035}{2}\right)^{2 \cdot 5} = \$3,568.33$$

8. Chase pays 2% compounded monthly.

$$A = 3000\left(1 + \frac{0.02}{12}\right)^{12 \cdot 5} = \$3,315.24$$

9. Wells Fargo pays 3.5% compounded continuously.

$$A = 3000\, e^{0.035 \cdot 5} = \$3,573.74$$

10. U.S. Bank pays 4% compounded daily.

$$A = 3000\left(1 + \frac{0.04}{365}\right)^{365 \cdot 5} = \$3,664.17$$

11. When the Burmese Python was accidentally introduced into the Florida everglades by stowing away on a cargo ship, its population exploded. There were said to be four snakes on board. If that was nine years ago, and the population was 3 times larger each successive year, how many snakes are there now?

$A = 4 \cdot 3^9 =$ **78,732 snakes**

12. In an attempt to make a little extra spending cash, you attempt to play the stock market and invest $500 in a penny stock. Unfortunately you haven't been getting too lucky, and your stock's value has steadily dropped by 12% each week. How much are your shares still worth after 6 weeks?

$A = 500(1-0.12)^6 =$ **$232.20**

Solve each equation.

13. $16 = 2^x$

$2^4 = 2^x$

$x = 4$

14. $9^x \cdot 3^{x-6} = 1$

$(3^2)^x \cdot 3^{x-6} = 3^0$

$3^{2x+x-6} = 3^0$

$3x - 6 = 0 \rightarrow x = 2$

15. $\frac{1}{6} = 6^{x^2}$

$6^{-1} = 6^{x^2}$

$x^2 = -1$

$x = \pm i$

16. $4^{2x+1} = 8^x$

$(2^2)^{2x+1} = (2^3)^x$

$2^{4x+2} = 2^{3x}$

$4x + 2 = 3x \rightarrow x = -2$

17. $\frac{1}{25} = 125^{2x}$

$5^{-2} = (5^3)^{2x}$

$-2 = 6x$

$x = -1/3$

18. $\sqrt{27} = 3^{-2x}$

$(3^3)^{1/2} = 3^{-2x}$

$3/2 = -2x$

$x = -3/4$

19. $49^{3x} = \left(\frac{1}{49}\right)^{x^2-4}$

$49^{3x} = (49^{-1})^{x^2-4}$

$3x = 4 - x^2$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0 \rightarrow x = -4, 1$

20. $\sqrt[3]{2} = 2^{x+1}$

$2^{1/3} = 2^{x+1}$

$x + 1 = 1/3$

$x = -2/3$

Solve for the unknown variable in each of the following interest rate scenarios.

21. $A = \$7{,}035.63$
$P = ?$
$r = 9\%$ compounded continuously
$t = 6$ years

$7035.63 = P \cdot e^{0.09 \cdot 6}$

$7035.63 = 1.716P$

$\boxed{P = \$4{,}100}$

22. $A = \$2{,}813.77$
$P = \$2{,}500$
$r = ?\%$ compounded semiannually
$t = 2$ years

$2813.77 = 2500\left(1 + \frac{r}{2}\right)^{2 \cdot 2}$

$1.125 = \left(1 + \frac{r}{2}\right)^4$ ← Take 4th root of both sides

$1.03 = 1 + \frac{r}{2}$

$\boxed{r = 0.06 = 6\%}$

23. You are presented with two options for a savings account in which you'd like to invest your money. They are both offering to pay you the same annual interest rate, but one is compounded quarterly while the other is compounded daily. Explain the difference between the two accounts, and state why one is a better choice than the other.

When interest is compounded continuously you are constantly earning interest on each previous moment's interest. This is more profitable than just earning interest on the previous quarter's balance.

24. The radioactive element thorium-91 has a half-life of 180 years. An artifact known to 632 years old contains 3 grams of thorium-91. How much of this 3 gram sample should still be radioactive today?

$A = 3 \cdot \left(\frac{1}{2}\right)^{632/180} = \boxed{0.26 \text{ grams}}$

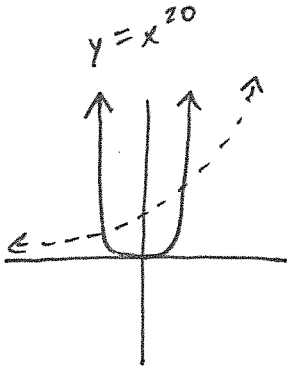
25. The radioactive element carbon-12 has a half-life of 23,000 years. A bone has been found that contains a 1.4 gram sample of carbon-12. If 0.35 grams are still radioactive, how old is this bone?

$0.35 = 1.4\left(\frac{1}{2}\right)^{t/23000}$

$0.25 = \left(\frac{1}{2}\right)^{t/23000}$ → $\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/23000}$

$\frac{t}{23000} = 2$ → $\boxed{t = 46000 \text{ years}}$

26. Compare and contrast the functions $f(x) = x^{20}$ and $g(x) = 2^x$.



Both increase without bound as x gets larger. As x gets smaller, only x^{20} increases. while 2^x approaches an asymptote: $y = 0$. x^{20} appears to grow faster at first, but 2^x will catch up.