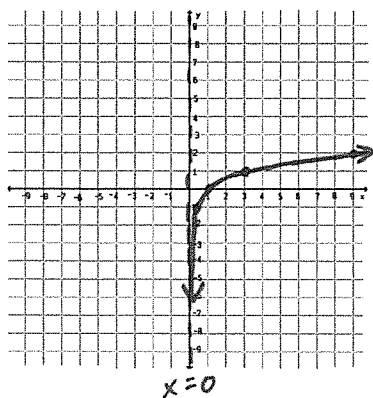


Practice Test 6  
Logarithms

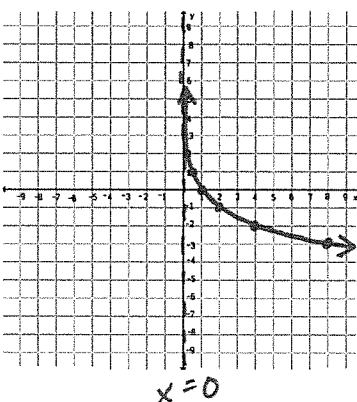
You may not use a calculator for this section of the test.

Graph the following functions. You must plot at least three clear points and any asymptotes with a dashed line.

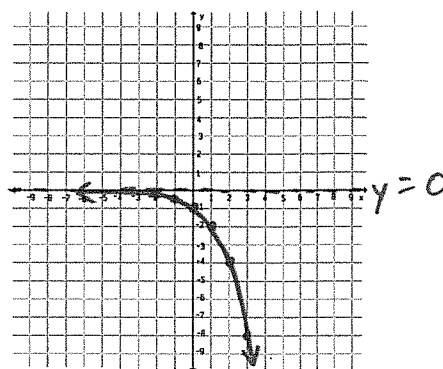
1.  $f(x) = \log_3 x$



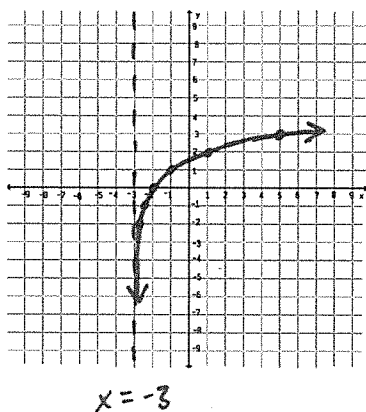
2.  $f(x) = \log_{\frac{1}{2}} x$



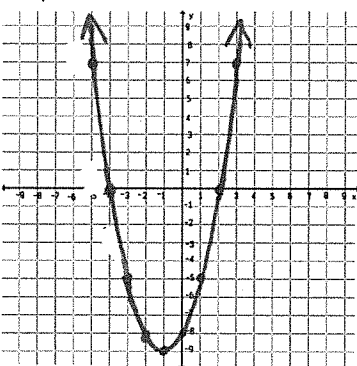
3.  $f(x) = -2^x$



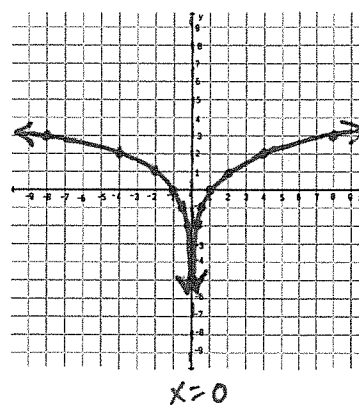
4.  $f(x) = \log_2(x + 3)$



5.  $f(x) = x^2 + 2x - 8$   
 $f(x) = (x+1)^2 - 9$



6.  $f(x) = \log_2|x|$



Rewrite each logarithmic equation as an exponential equation.

7.  $h = \log_5 w$

$5^h = w$

8.  $\log_a 13 = x^2$

$a^{x^2} = 13$

9.  $4 = \ln y$

$e^4 = y$

Rewrite each <sup>exponential</sup> logarithmic equation as an <sup>logarithmic</sup> exponential equation.

10.  $8^{10} = m$

$\log_8 m = 10$

11.  $z^3 = b$

$\log_z b = 3$

12.  $2 = 10^k$

$\log 2 = k$

13. Let  $f(x) = 7^x - 5$ . Find  $f^{-1}(x)$ .

$x = 7^y - 5$

$7^y = x + 5$

$y = \log_7(x + 5)$

$f^{-1}(x) = \log_7(x + 5)$

Condense each expression into one logarithm.

14.  $\log_3 x + \log_3 4$

$$\log_3 4x$$

Expand each expression.

16.  $\log_{\frac{2}{3}} xy^2$

$$\log_{\frac{2}{3}} x + 2 \log_{\frac{2}{3}} y$$

Solve each equation for  $x$ .

18.  $\log_x 3 = \frac{1}{2}$

$$x^{\frac{1}{2}} = 3$$

$$x = 3^2$$

$$\boxed{x = 9}$$

20.  $\log_2 x - \log_2(x-2) = 2$

$$\log_2 \frac{x}{x-2} = 2$$

$$\frac{x}{x-2} = 2^2$$

$$x = 4x - 8 \rightarrow \boxed{x = \frac{8}{3}}$$

Find the domain and range of each function.

22.  $f(x) = 3 + \log(x+4)$

$$x+4 > 0 \rightarrow x > -4$$

$$\boxed{\begin{array}{l} \text{Domain: } (-4, \infty) \\ \text{Range: } (-\infty, \infty) \end{array}}$$

24.  $f(x) = \log_5|x+2|$

$|x+2| > 0$   
This is always positive except when  $x+2=0 \rightarrow x=-2$

$$\boxed{\begin{array}{l} \text{Domain: } x \neq -2 \\ \text{Range: } (-\infty, \infty) \end{array}}$$

15.  $5 \ln a - 3 \ln b$

$$\ln \frac{a^5}{b^3}$$

17.  $\log\left(\frac{s}{t}\right)^p$

$$p \log s - p \log t$$

19.  $\log(x+3) = 1$

$$10^1 = x+3$$

$$\boxed{x = 7}$$

21.  $3 \ln x = \ln x$

$$\ln x^3 = \ln x$$

$$x^3 = x$$

$$x(x+1)(x-1) = 0$$

$$\cancel{x=0} \quad \cancel{x=-1} \quad \boxed{x=1}$$

23.  $f(x) = e^x + 2$  ← up 2

$$\boxed{\begin{array}{l} \text{Domain: } (-\infty, \infty) \\ \text{Range: } (2, \infty) \end{array}}$$

25.  $f(x) = \ln(x^2 - 4)$

$$x^2 - 4 > 0$$

$$x^2 > 4 \rightarrow x > 2 \text{ or } x < -2$$

$$\boxed{\begin{array}{l} \text{Domain: } (-\infty, -2) \cup (2, \infty) \\ \text{Range: } (-\infty, \infty) \end{array}}$$

You may use a calculator for this section of the test. All decimals can be rounded to the nearest hundredth.

Express each of the following with base  $\beta$ .

1.  $\log_3 4$

$$\frac{\log_{\beta} 4}{\log_{\beta} 3}$$

2.  $\log_{1.2} a$

$$\frac{\log_{\beta} a}{\log_{\beta} 1.2}$$

3.  $\ln 10$

$$\frac{\log_{\beta} 10}{\log_{\beta} e}$$

Evaluate each expression.

4.  $\log_2 11$

$$\frac{\log 11}{\log 2} = 3.46$$

5.  $\log 0.5$

$$-0.30$$

6.  $\log_8 \pi$

$$\frac{\log \pi}{\log 8} = 0.55$$

Solve each equation for  $x$ .

7.  $10^x = 17$

$$x = \log 17$$

$$\boxed{x = 1.23}$$

8.  $\frac{2}{3}e^x + 1 = \frac{3}{2}$

$$\frac{2}{3}e^x = \frac{1}{2} \rightarrow e^x = \frac{3}{4}$$

$$x = \ln\left(\frac{3}{4}\right) \rightarrow \boxed{x = -0.29}$$

9.  $2^x = 5^{x-4}$

$$\log_2 5^{x-4} = x$$

$$(x-4)\log_2 5 = x$$

$$(x-4)2.321 = x$$

$$\boxed{x = -7.03}$$

10.  $2^{x^2} = 4$

$$2^{x^2} = 2^2$$

$$x^2 = 2$$

$$\boxed{x = \pm\sqrt{2} \text{ or } \pm 1.41}$$

11. You've invested all of your savings into one stock. Fortunately for you, the stock price rises steadily at a rate of 15% per year. How long before your shares double in value?

$$2P = P(1+.15)^t \rightarrow 2 = 1.15^t \quad t = \log_{1.15} 2 = \boxed{4.96 \text{ years}}$$

12. If smallpox were rereleased by the Center for Disease Control, it is estimated that the number of infected individuals would triple each day. Beginning with just one case of smallpox, how many days would it take for the entire world - 7 billion people - to become infected?

$$7,000,000,000 = 1.3^t \quad t = \log_3 7000000000 \quad \boxed{t = 20.63 \text{ days}}$$

13. Uranium-242 has a half-life of 2.3 days. A body has been examined in a forensics lab, and it was found that 0.6 grams of a 1.7 grams sample of uranium-242 were still radioactive. How long has the body been dead?

$$0.6 = 1.7\left(\frac{1}{2}\right)^{\frac{t}{2.3}} \rightarrow 0.35 = \left(\frac{1}{2}\right)^{\frac{t}{2.3}} \rightarrow \frac{t}{2.3} = \log_{\frac{1}{2}} 0.35$$

$$t = 2.3 \log_{\frac{1}{2}} 0.35 = \boxed{3.48 \text{ days}}$$