

Practice Test 7
Counting and Probability

Calculate the following.

$$1. {}_9P_2 = \frac{9!}{7!} = 9 \cdot 8$$

$$= \boxed{72}$$

$$2. {}_8C_1 = \frac{8!}{7! \cdot 1!}$$

$$= \boxed{8}$$

$$3. {}_5P_3 = \frac{5!}{2!} = 5 \cdot 4 \cdot 3$$

$$= \boxed{60}$$

$$4. {}_{10}P_{20}$$

$$= \boxed{0}$$

$$5. {}_9C_4 = \frac{9!}{5! \cdot 4!} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$= \boxed{126}$$

$$6. {}_{100}C_0 = \frac{100!}{100! \cdot 0!}$$

$$= \boxed{1}$$

Identify each scenario as a permutation or a combination. You do not need to calculate anything

7. The number of different ways the horses in the Kentucky Derby could finish first, second, and third.

Permutation

8. The number of different sequences of numbers you could try to open the lock on your friend's locker.

Permutation

9. The number of ways we could split the class into two teams.

Combination

Simplify each expression. You do not need to calculate anything.

$$10. \frac{43!}{43 \cdot 42} = \boxed{41!}$$

$$11. 20 \cdot 18! \cdot 19 = \boxed{20!}$$

$$12. n!(n+1) = \boxed{(n+1)!}$$

$$13. (n^3 - n)(n-2)! = n(n^2 - 1)(n-2)!$$

$$= n(n+1)(n-1)(n-2)! = \boxed{(n+1)!}$$

14. Suppose ${}_nC_5 = 1$. What must n equal? Explain.

$$\boxed{n=5}$$

There is only one way to choose a team of 5 out of a group of 5 people.

15. How many ways can a club with 10 students choose a president, vice president, and a treasurer?

$${}_{10}P_3 = \frac{10!}{7!} = 10 \cdot 9 \cdot 8 = \boxed{720}$$

16. A group of eight people have shown up to an open interview. If there are two open positions (both the same position), in how many ways can the new hires be chosen?

$${}^8C_2 = \frac{8!}{6! \cdot 2!} = \frac{8 \cdot 7}{2 \cdot 1} = \boxed{28}$$

17. A restaurant menu offers three different appetizers, five main courses, and two desserts. In how many ways could you order dinner if you'd like one of each?

$$3 \cdot 5 \cdot 2 = \boxed{30}$$

18. a) How many words (including nonsense words) can be made by rearranging all of the letters in the word SOCK?

$${}_4P_4 = \frac{4!}{0!} = 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{24}$$

b) How many words (including nonsense words) can be made by rearranging all of the letters in the word SOCKS?

Since you can swap the two S's.

$$\frac{{}_5P_5}{2} = \frac{5!}{2} = \boxed{60}$$

For questions 21 and 22, identify each set of events as dependent or independent.

19. A coin is flipped multiple times in a row.

Independent

20. An object is randomly pulled from a mystery bag. This object is set aside, and another object is selected from the bag.

Dependent

21. Suppose you roll an ordinary 6-sided die once. Let A be the event that this one roll comes up even, and let B be the event that this one roll comes up odd. Find the following.

a) $P(A) = \frac{1}{2}$

b) $P(A \cup B) = 1$

c) $P(A \cap B) = 0$

can't be even and odd

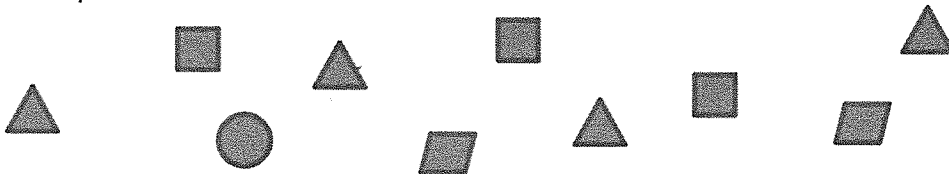
d) $P(B|A) = 0$

Given that it's even, it must not be odd.

22. You've just flipped a coin one thousand times and it has come up heads every time. What is the theoretical probability that it comes up heads again on the next flip?

$$\boxed{\frac{1}{2}}$$

For questions 23-25, you will be reaching into a bag and choosing randomly from the following selection of shapes. Write your answer as a reduced fraction.



23. You select one shape at random. What is the probability that it is...

a) Four-sided? $\frac{5}{10} = \boxed{\frac{1}{2}}$

b) A triangle or a circle? $\frac{5}{10} = \boxed{\frac{1}{2}}$

c) A square or not a square? $\boxed{1}$

d) Not a circle? $\boxed{\frac{9}{10}}$

24. You select one shape at random, set it aside, then select a second shape at random. What is the probability that...

a) Both shapes are triangles?

$$\frac{4}{10} \cdot \frac{3}{9} = \boxed{\frac{2}{15}}$$

b) The first is a square and the second is four-sided?

$$\frac{3}{10} \cdot \frac{4}{9} = \boxed{\frac{2}{15}}$$

c) The first shape is a circle and the second is not a circle?

$$\frac{1}{10} \cdot \frac{9}{9} = \boxed{\frac{1}{10}}$$

d) Both shapes are circles?

$$\frac{1}{10} \cdot \frac{0}{9} = \boxed{0}$$

25. You select one shape at random, put it back into the bag, then select a second shape at random. What is the probability that...

a) Both shapes are triangles?

$$\frac{4}{10} \cdot \frac{4}{10} = \boxed{\frac{4}{25}}$$

b) The first is a square and the second is four-sided?

$$\frac{3}{10} \cdot \frac{5}{10} = \boxed{\frac{3}{20}}$$

c) The first shape is a circle and the second is not a circle?

$$\frac{1}{10} \cdot \frac{9}{10} = \boxed{\frac{9}{100}}$$

d) Both shapes are circles?

$$\frac{1}{10} \cdot \frac{1}{10} = \boxed{\frac{1}{100}}$$