Practice Test
Sequences and Series

Key

Determine if the following sequences are arithmetic, geometric, or neither. If the sequence is arithmetic, find d. If the sequence is geometric, find r.

1. $-1, 3, 7, 11, \ldots$ (+4, +4, +4)

arithmetic
$d = 4$

2. 1, 4, 9, 16, …

neither

3. $a_n = 7 - n$ $(6, 5, 4, 3, \ldots)$ (−1, −1, −1)

arithmetic
$d = -1$ (the slope)

4. $5, -10, 20, -40, \ldots$ (×−2, ×−2, ×−2)

geometric
$r = -2$

5. $a_n = 4n^3 - 2$

neither

6. $a_n = 7\left(\frac{1}{2}\right)^n$ $\left(\frac{7}{2}, \frac{7}{4}, \frac{7}{8}, \ldots\right)$ (×½, ×½)

geometric
$r = \frac{1}{2}$

Both of the sequences described below are <u>arithmetic</u>. For each one, find a formula for the n^{th} term. Then use your formula to calculate the 30th term.

7. $a_2 = 0 \quad d = -2$

$a_1 = 2$

$$\boxed{a_n = 2 + -2(n-1) \text{ or } a_n = 4 - 2n}$$

$a_{30} = 4 - 2(30) = \boxed{-56}$

8. $a_3 = 8 \quad a_7 = 32$

$d = \frac{32-8}{7-3} = \frac{24}{4} = 6$

$a_1 = 8 - 2(6) = -4$

$$\boxed{a_n = -4 + 6(n-1) \text{ or } a_n = -10 + 6n}$$

$a_{30} = -10 + 6(30) = \boxed{170}$

Both of the sequences described below are <u>geometric</u>. For each one, find a formula for the n^{th} term. Then use your formula to calculate the 7th term.

9. $a_1 = 6 \quad r = 1.5$

$$\boxed{a_n = 6(1.5)^{n-1}}$$

$a_7 = 6(1.5)^{7-1}$

$= \boxed{68.34}$

10. $a_3 = 24 \quad a_4 = 48$

$r = \frac{48}{24} = 2 \quad a_1 = \frac{24}{2^2} = 6$

$$\boxed{a_n = 6(2)^{n-1}}$$

$a_7 = 6(2)^{7-1} = \boxed{384}$

11. Describe why the formula $S_n = \frac{n(a_1+a_n)}{2}$ will not produce the correct answer for the sum $\sum_{k=1}^{50}(2k^2+1)$.

The formula only works for arithmetic series. Check the first few terms: 3, 9, 19, ... it is not arithmetic. (k^2 messes it up.)

Calculate the following and use them for questions 15–22 when necessary.

These are useful for # 18 + 22.

12. $\sum_{k=1}^{12} 1 = \boxed{12}$

13. $\sum_{k=1}^{12} k = \frac{12(12+1)}{2} = \boxed{78}$

14. $\sum_{k=1}^{12} k^2 = \frac{12(12+1)(12\cdot 2+1)}{6} = \boxed{650}$

Calculate the following.

15. $\sum_{k=1}^{10} 2\left(\frac{3}{2}\right)^k$ (geometric formula)

$\frac{3(1-(\frac{3}{2})^{10})}{1-\frac{3}{2}} = \boxed{340}$

16. $\sum_{k=-28}^{-1} (3k+7)$ (arithmetic formula)

$\frac{28(-77+4)}{2} = \boxed{-1{,}022}$

17. $\sum_{k=3}^{\infty} 8\left(\frac{1}{2}\right)^k$ (geometric formula)

equals 0!!

$\frac{1(1-(\frac{1}{2})^{\infty})}{1-\frac{1}{2}} = \frac{1}{1-\frac{1}{2}} = \boxed{2}$

18. $\sum_{k=1}^{12} (2k^2 - 13k)$ (use #13 + 14)

$= 2\sum_{k=1}^{12} k^2 - 13\sum_{k=1}^{12} k$

$= 2(650) - 13(78) = \boxed{286}$

19. $\sum_{k=2}^{40} \left(\frac{k}{2} - 5\right)$ (arithmetic formula)

$\frac{39(-4+15)}{2} = \boxed{214.5}$

20. $\sum_{k=1}^{\infty} \frac{2}{3}k = \frac{2}{3} + \frac{4}{3} + \frac{6}{3} + \frac{8}{3} + \cdots = \boxed{\infty}$

clearly keeps growing

21. $\sum_{k=7}^{8} (-1)^k k$

$= (-1)^7 \cdot 7 + (-1)^8 \cdot 8$

$= -7 + 8 = \boxed{1}$

22. $\sum_{k=1}^{12} (k-9)(k+1)$ (use # 12, 13, + 14)

$= \sum_{k=1}^{12} (k^2 - 8k - 9)$

$= 650 - 8(78) - 9(12) = \boxed{-82}$

23. If the sum $\sum_{k=0}^{21} k^4$ is equal to 917,147, what must $\sum_{k=-21}^{21} k^4$ be equal to? Explain.

First notice that $\sum_{k=-21}^{0} k^4 = \sum_{k=0}^{21} k^4$ since $(-21)^4 = 21^4$, $(-20)^4 = 20^4$, etc.

So we're really just adding the same sum twice...

$2(917{,}147) = \boxed{1{,}834{,}294}$

24. How many terms are listed in the following sequence?

5, 8, 11, 14, …, 74

$74 = 5 + 3(n-1)$

$69 = 3(n-1) \rightarrow n-1 = 23$

$\boxed{n = 24}$

Rewrite each of the following sums using sigma notation.

25. 10 + 16 + 22 + 28 + 34

$(10+6(0)) + (10+6(1)) + \cdots + (10+6(4))$

$\boxed{= \sum_{k=0}^{4} (10+6k)}$

26. 3 − 6 + 12 − 24 + 48 − 96

$3(-2)^0 + 3(-2)^1 + 3(-2)^2 + \cdots + 3(-2)^5$

$\boxed{= \sum_{k=0}^{5} 3(-2)^k}$

27. 2 + 5 + 10 + 17 + 26 + 37 + 50

$(1^2+1) + (2^2+1) + (3^2+1) + \cdots + (7^2+1)$

$\boxed{= \sum_{k=1}^{7} (k^2+1)}$

28. $\frac{3}{1} + \frac{3}{2} + \frac{3}{3} + \frac{3}{4} + \frac{3}{5} + \frac{3}{6}$

denominators go up 1 each time.

$\boxed{= \sum_{k=1}^{6} \frac{3}{k}}$

Extra credit thoughts:

If $a_1, a_2, a_3, \ldots$ is an arithmetic sequences, you already know the formula for the partial sums S_n and S_k.

Can you use these two formulas to produce one for $\sum_{i=k}^{n} a_i$?

$S_n = \frac{n(a_1+a_n)}{2}$

$S_k = \frac{k(a_1+a_k)}{2}$

How can we use these to get this?

Could an infinite arithmetic series (besides the trivial series 0 + 0 + 0 + …) ever equal anything other than ∞ or −∞? Why or why not?

What must be true about i if $\sum_{k=-n}^{n} k^i = 0$ for any value of n?

Pick a small value of n (maybe n=2) and test for a few different values of i.