Basic Parabolas Review

Put the parabolas in vertex form and state the vertex. Then tell whether the vertex is the highest point or lowest point on the parabola.

1. $x^2 - 4x + 9 + 4 - 4$

$(x-2)^2+5$

vertex: $\boxed{(2,5)}$

Lowest point since parabola opens up

2. $-x^2 + 5x$

$-(x^2-5x)$

$= -(x^2-5x+6.25-6.25)$

$= -(x-2.5)^2+6.25$

vertex: $\boxed{(2.5, 6.25)}$ High point ... opens down

3. $2x^2 + 6x + 7$

$2(x^2+3x)+7$

$2(x^2+3x+2.25-2.25)+7$

$2(x+1.5)^2-4.5+7$

$=2(x+1.5)^2+2.5$

vertex: $\boxed{(-1.5, 2.5)}$ Low point

The parabolas below define the height of a projectile over time. For each one, find the time(s) when the projectile is on the ground, and find its maximum height.

4. $-4x^2 + 20x$

$0=-4x^2+20x$

$0=-4x(x-5)$

On ground: $\boxed{0 \text{ \& } 5 \text{ seconds}}$

x-value of vertex $= \frac{-b}{2a} = \frac{-20}{2(-4)} = 2.5$

$y=-4(2.5)^2+20(2.5) = 25$

max height = $\boxed{25 \text{ feet}}$

5. $-10x^2 + 15x + 5$

$0=-5(2x^2-3x-1)$

$x = \frac{3 \pm \sqrt{9-4(-1)(2)}}{2(2)} = \frac{3 \pm \sqrt{17}}{4}$

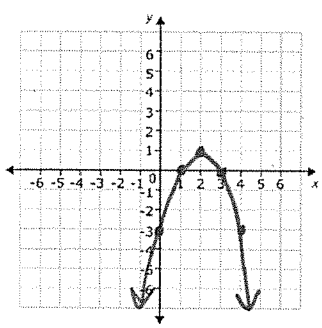
$\approx 1.78, -0.28$ On ground: $\boxed{1.78 \text{ sec}}$

$\frac{-b}{2a} = \frac{3}{2(2)} = 0.75$

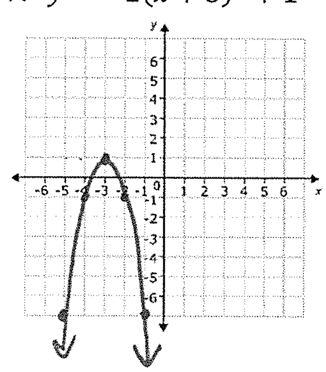
$-10(0.75)^2+15(0.75)+5 = \boxed{10.625 \text{ ft}}$

Graph each parabola. Plot at least five points accurately.

6. $y = -x^2 + 4x - 3 = -(x-3)(x-1)$



7. $y = -2(x+3)^2 + 1$



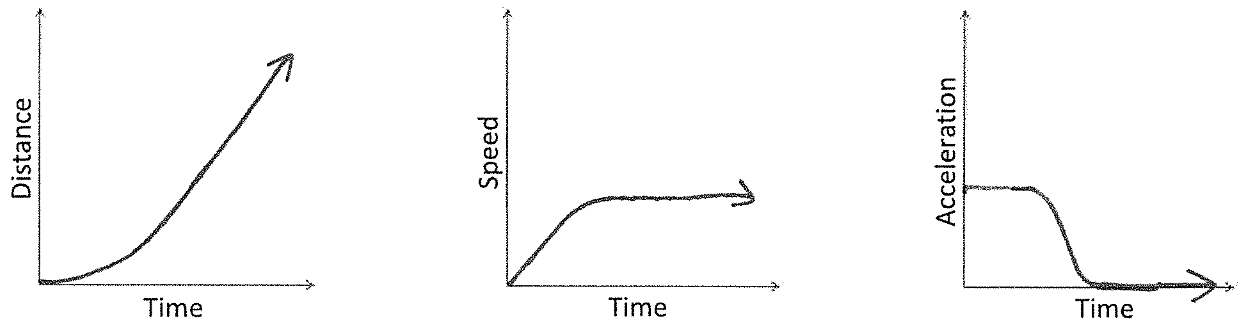
Distance, Velocity, and Acceleration

1. List the equations relating distance to velocity and velocity to acceleration.

$d = vt + d_i$ $\qquad$ $v = at + v_i$

Make quick sketches that represent the situation.

2. For the first few seconds of a race, a runner accelerates at a constant rate. Once he hits top speed, he runs with a constant velocity.



3. Beginning from a stop, you accelerate at a constant rate of 4 meters/second^2. How fast are you going after 7 seconds?

$v = at + v_i$

$v = 4(7) + 0$

$\boxed{v = 28 \text{ m/sec}}$

4. If you drive at a constant rate for 6 hours and cover a total distance of 438 miles, how fast were you driving?

$d = vt + d_i$

$438 = v(6) + 0$

$\boxed{v = 73 \text{ mi/hr}}$

5. While approaching a red light you apply the brakes and decelerate at a rate of 6.1 meters/second^2. If you were initially driving at 28 m/s, how long will it take you to come to a stop?

$v = at + v_i$

$0 = -6.1t + 28$

$\boxed{t = 4.59 \text{ sec.}}$

6. Beginning from a stop a sprinter needs 1.5 seconds to reach their top speed of 10.3 meters/second. What is there average acceleration over the 1.5 seconds?

$v = at + v_i$

$10.3 = a(1.5) + 0$

$\boxed{a = 6.87 \text{ m/sec}^2}$

Freefall Without Angles

A game is played among a very stupid group of friends in Mr. Martin's home state of Kentucky. One of them fires an arrow straight up into the air. The winner is the one that waits the longest to run off to the side. One particular arrow is fired from a height of 6 feet with an initial velocity of 94 feet/second.

1. Based on the information above, what do the following variables equal?

$h_i = 6$ ft $\qquad v_i = 94$ ft/sec $\qquad a = -32$ ft/sec^2

2. Using your answer to question 1, write an equation relating the height of the arrow above ground in feet, *h*, to the time since the bullet was fired in seconds, *t*.

$$h(t) = -16t^2 + 94t + 6$$

3. How high above ground will the arrow be 3 seconds after it was fired?

$$h(3) = -16(3)^2 + 94(3) + 6 = \boxed{144 \text{ ft}}$$

4. How high above the ground will the arrow be 8 seconds after it was fired? Explain your answer.

$$h(8) = -16(8)^2 + 94(8) + 6 = -266 \quad \boxed{0 \text{ ft}}$$

The arrow won't continue to go below the ground.

5. When will the arrow be 80 feet in the air? Explain why there are two different answers.

$$80 = -16t^2 + 94t + 6$$
$$0 = -16t^2 + 94 - 74$$
$$t = \frac{-94 \pm \sqrt{94^2 - 4(16)(74)}}{2(-16)} \approx \boxed{0.94,\ 4.94 \text{ seconds}}$$

on way up   on way down

6. How long do the friends have before the arrow reaches the height of their heads (about 6 feet)?

$$6 = -16t^2 + 94t + 6$$
$$0 = -16t^2 + 94t \rightarrow 0 = -2t(8t - 47)$$
$$t = 0,\ 5.875 \text{ sec}$$

$\boxed{\text{About } 5.88 \text{ seconds}}$

7. When will the arrow hit the ground?

$$0 = -16t^2 + 94t + 6$$
$$t = \frac{-94 \pm \sqrt{94^2 - 4(6)(-16)}}{2(-16)} \approx -0.06,\ 5.94 \quad \boxed{5.94 \text{ seconds}}$$

Trigonometry

Calculate each of the following to the nearest thousandth.

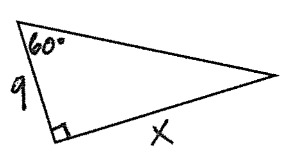
1. sin(18) = 0.309

2. $\cos^{-1}(0.6)$ = 53.130

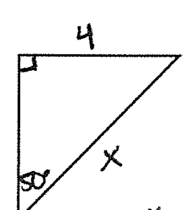
3. sin(80) 0.985

4. $\tan^{-1}(1)$ = 45

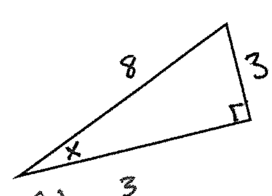
Find the missing measurement.

5. 

$$\tan(60) = \frac{x}{9}$$
$$x = 9\tan(60) = \boxed{15.588}$$

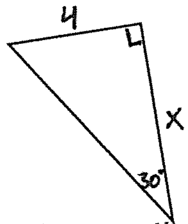
6. 

$$\sin(50) = \frac{4}{x}$$
$$x = \frac{4}{\sin(50)} = \boxed{5.222}$$

7. 

$$\sin(x) = \frac{3}{8}$$
$$x = \sin^{-1}\left(\frac{3}{8}\right) = \boxed{22.024}$$

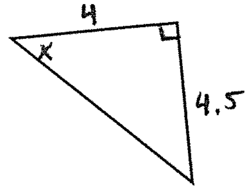
8.



$\tan(30) = \frac{4}{x}$

$x = \frac{4}{\tan(30)} = \boxed{6.928}$

9.

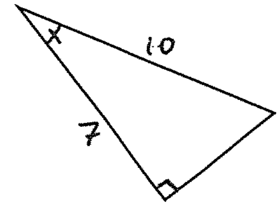


$\tan(x) = \frac{4.5}{4}$

$x = \tan^{-1}\left(\frac{4.5}{4}\right)$

$\boxed{= 48.366}$

10.



$\cos(x) = \frac{7}{10}$

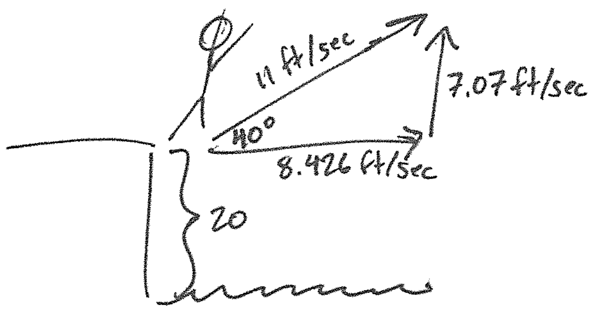
$x = \cos^{-1}\left(\frac{7}{10}\right)$

$\boxed{x = 45.573}$

Freefall With Angles

For each situation below, draw a picture and label the initial height, initial velocity, horizontal velocity, initial vertical velocity, and angle of trajection.

1. A diver runs and jumps off of a platform at an angle of 40 degrees to the water with a velocity of 11 feet per second. The platform is elevated 20 feet above the pool.



a) Write two equations: one that relates the diver's horizontal distance from the platform to time, and another that relates his height off the ground to time.

$h(t) = -16t^2 + 7.07t + 20$ $\qquad$ $d = 8.426t$

b) How far from the edge of the platform will the diver be after 1 second?

$d = 8.426(1) = \boxed{8.426 \text{ ft}}$

c) What is the maximum height above the water the diver will reach?

$t_{vertex} = \frac{-b}{2a} = \frac{-7.07}{2(-16)} = 0.221 \text{ sec}$ $\qquad$ $h(0.221) = -16(0.221)^2 + 7.07(0.221) + 20$

$\boxed{= 20.78 \text{ ft}}$

d) How far from the edge of the platform will the diver be when he reaches the water?

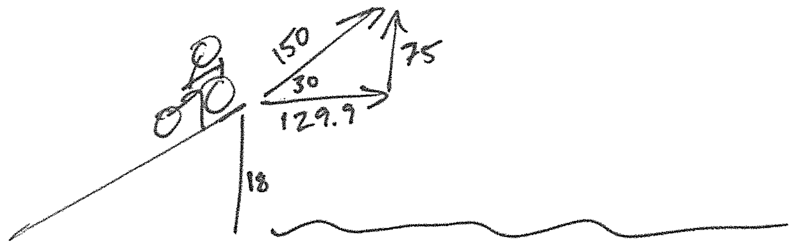
$0 = -16t^2 + 7.07t + 20$

$t = \frac{-7.07 \pm \sqrt{7.07^2 - 4(20)(-16)}}{2(-16)} \approx -0.92, 1.36$

$d = 8.426(1.36)$

$= \boxed{11.459 \text{ ft}}$

2. A stuntman is planning on jumping the Mississippi River on his motorcycle. The end of the ramp he has set up is 18 feet off of the ground, and it will launch him at an angle of 30 degrees. He is capable of hitting a speed of 150 feet per second on takeoff. There is no landing ramp on the other side of the buses.



a) Write two equations: one that relates the stuntman's horizontal distance from the ramp to time, and another that relates his height off the ground to time.

$h(t) = -16t^2 + 75t + 18$ $\quad\quad$ $d = 129.9t$

b) How far out from the ramp will the stuntman be when he hits his maximum height?

$t_{vertex} = \frac{-b}{2a} = \frac{-75}{2(-16)} = 2.34$ sec $\quad\quad$ $d = 129.9(2.34) = \boxed{304.45 \text{ ft}}$

c) The other side of the Mississippi River is 800 feet away. Will the stuntman make it? (Hint: first find when he will reach ground level.)

$0 = -16t^2 + 75t + 18$

$t = \frac{-75 \pm \sqrt{75^2 - 4(-16)(18)}}{2(-16)}$

$t = -0.23, 4.92$

in the air for this long

$d = 129.9(4.92)$

$= \boxed{639.11 \text{ ft}}$

Not even close! Bad idea!

Definition of *i*

Simplify each number and write with i notation if necessary.

1. $\sqrt{-4} = 2i$

2. $-\sqrt{8} = -2\sqrt{2}$

3. $\sqrt{-2} = i\sqrt{2}$

4. $2i + \sqrt{-9} = 2i + 3i$ $\boxed{= 5i}$

5. $-5\sqrt{-1} = \boxed{-5i}$

6. $3\sqrt{-50} = 3\sqrt{-25 \cdot 2}$ $= \boxed{15i\sqrt{2}}$

Simplify.

7. $i^2 = \boxed{-1}$

8. $-i^3 = -(-i) = \boxed{i}$

9. $i^{40} = (i^4)^{10} = 1^{10}$ $= \boxed{1}$

10. $i^{202} = i^{200} \cdot i^2$

$= 1 \cdot -1 = \boxed{-1}$

11. $(i - i^3)^2$

$= (i - (-i))^2$

$= (2i)^2 = \boxed{-4}$

12. $i\sqrt{-1}$

$= i \cdot i = \boxed{-1}$

$+/-/\times/\div$ Complex Numbers

Perform the indicated operation. Right your final answers in $a + bi$ form.

1. $(1 + 8i) - (2 - i)$

$= \boxed{-1 + 9i}$

2. $-2i + (1 + i)$

$= \boxed{1 - i}$

3. $\frac{3-2i}{i} \cdot \frac{i}{i} = \frac{3i - 2i^2}{-1}$

$= \boxed{-2 - 3i}$

4. $5i(1 + 4i) = 5i + 20i^2$

$= \boxed{-20 + 5i}$

5. $\frac{4+2i}{1+i} \cdot \frac{1-i}{1-i}$

$= \frac{4 + 2i - 4i - 2i^2}{2}$

$= \frac{6 - 2i}{2} = \boxed{3 - i}$

6. $(5 - 2i)(5 + 2i)$

$= 25 - 10i + 10i - 4i^2$

$= \boxed{29}$

(Shortcut is: $(a+bi)(a-bi) = a^2 + b^2$)

7. $\frac{5-2i}{2-i} \cdot \frac{2+i}{2+i}$

$= \frac{10 - 4i + 5i - 2i^2}{5}$

$= \frac{12 + i}{5} = \boxed{\frac{12}{5} + \frac{1}{5}i}$

8. $(3 - 4i)(1 + i)$

$= 3 - 4i + 3i - 4i^2$

$= \boxed{7 - i}$

Complex Roots

Find polynomials with real coefficients that have the following roots.

1. $x = 2i$, $-2i$

$(x-2i)(x+2i)$

$= x^2 - 2ix + 2ix - 4i^2$

$= \boxed{x^2+4}$

2. $x = -8$

$\boxed{x+8}$

3. $x = 3 - i$, $3+i$

$(x-(3-i))(x-(3+i))$

$= x^2 - (3-i)x - (3+i)x + (3-i)(3+i)$

$= \boxed{x^2 - 6x + 10}$

4. $x = 2$ and $x = -3i$, $3i$

$(x-2)(x-3i)(x+3i)$

$= (x-2)(x^2+9)$

$= \boxed{x^3 - 2x^2 + 9x - 18}$

Find all roots, both real and complex, of the following functions.

5. $f(x) = x^2 + 4x + 7$

$0 = x^2 + 4x + 7$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(7)}}{2(1)}$

$= \dfrac{-4 \pm \sqrt{-12}}{2} = \boxed{-2 + i\sqrt{3},\ -2 - i\sqrt{3}}$

6. $f(x) = x^2 + 9$

$0 = x^2 + 9$

$x^2 = -9$

$x = \pm\sqrt{-9} = \boxed{\pm 3i}$

7. $f(x) = x^3 + 4x$

$0 = x(x^2+4)$

$x = 0 \quad x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$x = \pm 2i$

$\boxed{x = 0,\ 2i,\ -2i}$

8. $f(x) = x^3 - 5x^2 - 6x$

$0 = x(x^2 - 5x - 6)$

$0 = x(x-6)(x+1)$

$\boxed{x = 0,\ 6,\ -1}$

Composition of Functions

For questions 1 – 6, let $f(x) = 2x^2 - 1$, $g(x) = x$, and $h(x) = x + 3$.

1. Find $f(h(-1))$.

$= f(2) = \boxed{7}$

2. Find $f(g(f(2)))$.

$= f(g(7)) = f(7) = \boxed{97}$

3. Find an expression for $(h \circ f)(x)$.

$= h(2x^2+1)$

$= (2x^2+1)+3$

$= \boxed{2x^2+4}$

4. Find an expression for $h(g(h(g(g(x)))))$.

$= h(h(x))$

$= h(x+3) = (x+3)+3$

$= \boxed{x+6}$

5. Find an expression for $(f \circ h)(x)$.

$= f(x+3)$

$= 2(x+3)^2 - 1 = \boxed{2x^2+12x+17}$

6. Find an expression for $f(f(x))$.

$= f(2x^2-1) = 2(2x^2-1)^2 - 1$

$= \boxed{8x^4 - 8x^2 + 1}$

Use the information below to evaluate each expression in questions 7 – 9.

x	$f(x)$
3	7
2	1
–1	1
0	2
–5	3

x	$h(x)$
6	–2
–2	–4
1	2
7	0
3	3

7. $(f \circ h)(7)$

$= f(0) = \boxed{2}$

8. $h(f(f(-5)))$

$= h(f(3))$

$= h(7) = \boxed{0}$

9. $(f \circ h)(1)$

$= f(2) = \boxed{1}$

Calculating Inverses

Calculate each inverse function.

1. $f(x) = 3x$

$x = 3y$

$y = \frac{x}{3}$

$\boxed{f^{-1}(x) = \frac{x}{3}}$

2. $f(x) = 2 - x$

$x = 2 - y$

$y = 2 - x$

$\boxed{f^{-1}(x) = 2 - x}$

3. $f(x) = \frac{2x+5}{3}$

$x = \frac{2y+5}{3}$

$2y+5 = 3x$

$y = \frac{3x-5}{2}$

$\boxed{f^{-1}(x) = \frac{3x-5}{2}}$

4. $f(x) = \frac{2}{x-5}$

$x = \frac{2}{y-5}$

$(y-5)x = 2$

$y-5 = \frac{2}{x}$

$\boxed{f^{-1}(x) = \frac{2}{x} + 5}$

5. $f(x) = x^3 - 5$

$x = y^3 - 5$

$y^3 = x + 5$

$\boxed{y = \sqrt[3]{x+5}}$

6. $f(x) = 2 - \sqrt[3]{x+1}$

$x = 2 - \sqrt[3]{y+1}$

$2 - x = \sqrt[3]{y+1}$

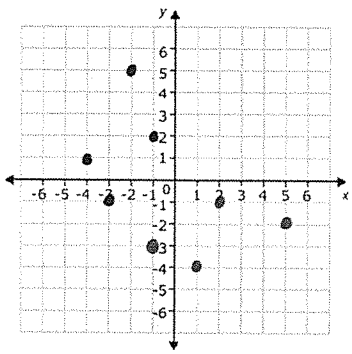
$y + 1 = (2-x)^3$

$\boxed{f^{-1}(x) = (2-x)^3 - 1}$

Graphing Inverses

If the functions below have an inverse function, graph it on the same plane as the original function. If an inverse function does not exist, write "not one-to-one."

1.



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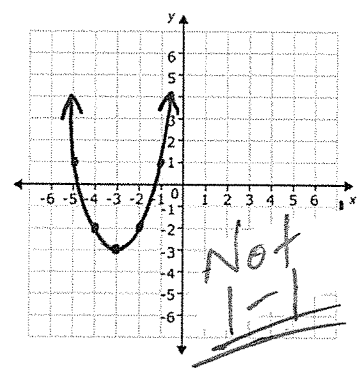
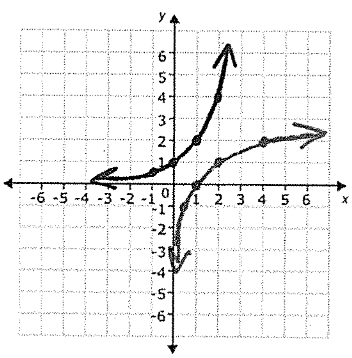


Not 1-1

3.



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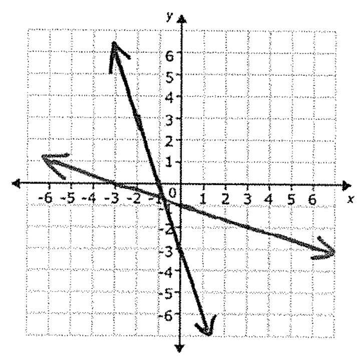

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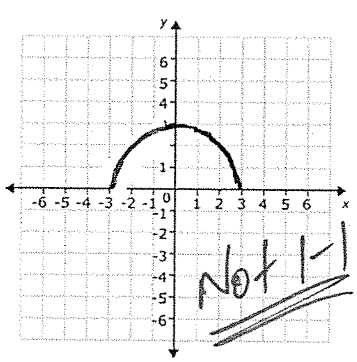

Not 1-1

6.