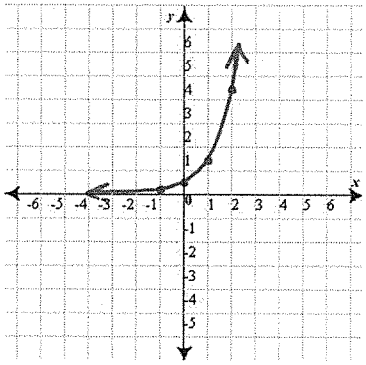
Basic Exponentials Review

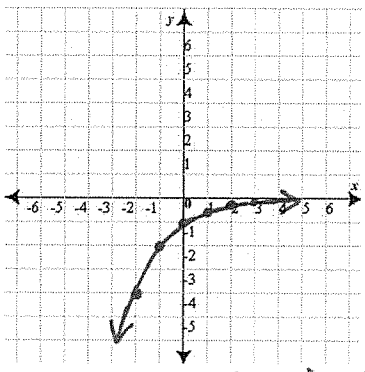
Graph each exponential and state the domain, range, and equation of the asymptote.

1. $f(x) = \frac{1}{2} \cdot 3^x$



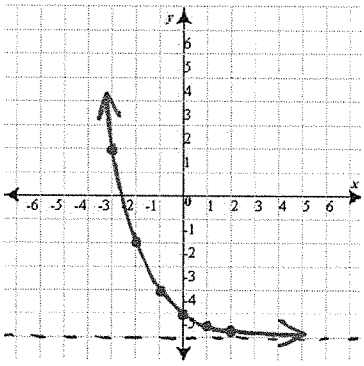
D: $(-\infty, \infty)$ R: $(0, \infty)$ As: $y = 0$

2. $y = -\left(\frac{1}{2}\right)^x$



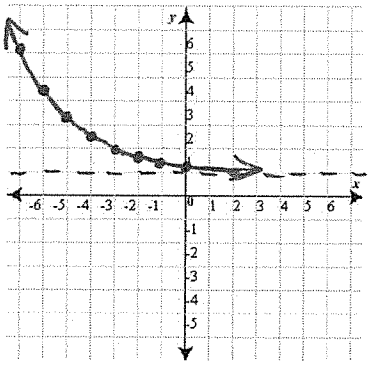
D: $(-\infty, \infty)$ R: $(-\infty, 0)$ As: $y = 0$

3. $f(x) = 2^{-x} - 6$



D: $(-\infty, \infty)$ R: $(-6, \infty)$ As: $y = -6$

4. $f(x) = \left(\frac{2}{3}\right)^{x+3} + 1$



D: $(-\infty, \infty)$ R: $(1, \infty)$ As: $y = 1$

Solve each exponential equation.

5. $2^x = 4^{x-1}$

$2^x = (2^2)^{x-1}$

$2^x = 2^{2x-2}$

$x = 2x - 2$

$\boxed{x = 2}$

6. $5^{x+2} = \left(\frac{1}{5}\right)^x$

$5^{x+2} = (5^{-1})^x$

$5^{x+2} = 5^{-x}$

$x + 2 = -x$

$\boxed{x = -1}$

7. $3 \cdot 9^x = \left(\frac{1}{27}\right)^x$

$3 \cdot (3^2)^x = (3^{-3})^x$

$3 \cdot 3^{2x} = 3^{-3x}$

$3^{2x+1} = 3^{-3x}$

$2x + 1 = -3x$

$\boxed{x = -\frac{1}{5}}$

Compound Interest and Other Word Problems

Suppose you invest $1,000 in the four savings accounts described below. How much money would be in each account after 11 years?

1. Bank of America pays 5% compounded annually.

$A = 1000(1.05)^{11} = \$1,710.34$

2. Chase pays 4% compounded quarterly.

$A = 1000\left(1 + \frac{0.04}{4}\right)^{11 \cdot 4} = \$1,549.32$

3. Wells Fargo pays 5% compounded continuously.

$A = 1000e^{0.05 \cdot 11} = \$1,733.25$

4. U.S. Bank pays 4% compounded daily.

$A = 1000\left(1 + \frac{0.04}{365}\right)^{11 \cdot 365} = \$1,570.04$

5. On January 1st there are three people diagnosed with a mysterious disease. How many infected individuals will there be by the end of December if the number of cases doubles each month?

$A = 3(2)^{12} = 12,288 \text{ people}$

6. Niobium-13 has a half-life of 23 days. How much of a 5 gram sample of niobium-13 will still be radioactive after 60 days?

$A = 5\left(\frac{1}{2}\right)^{\frac{60}{23}} = 0.82 \text{ grams}$

7. The radioactive element thorium-104 has a half-life of 13 years. A forensic detective has just found a bone belonging to a homicide victim that contains 10 grams of thorium-104. If 1.25 grams of the 10 grams of thorium-104 is still radioactive, how long ago did the victim die?

$\frac{1.25}{10} = \frac{10\left(\frac{1}{2}\right)^{\frac{t}{13}}}{10} \rightarrow 0.125 = \left(\frac{1}{2}\right)^{\frac{t}{13}}$

$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{\frac{t}{13}} \rightarrow \frac{t}{13} = 3$

$t = 39 \text{ years}$

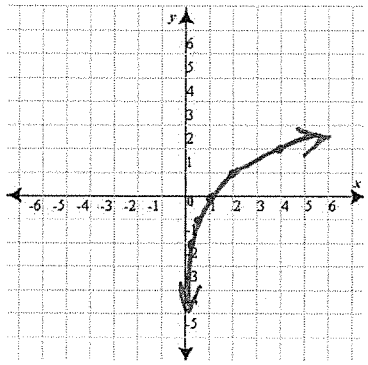
Calculating and Graphing Logarithms

Calculate.

1. $\log_3 \frac{1}{9} = -2$
2. $\log_2 1 = 0$
3. $\log_{100} 10 = \frac{1}{2}$
4. $\log_3 81 = 4$
5. $\log_4 \frac{1}{2} = -\frac{1}{2}$
6. $\log_7 \sqrt[5]{7} = \frac{1}{5}$

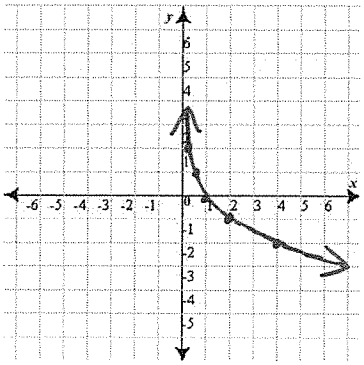
Graph each logarithm and state the domain, range, and equation of the asymptote.

1. $f(x) = \log_2 x$



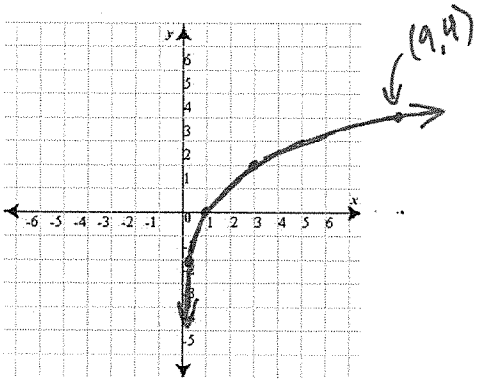
D: $(0,\infty)$ R: $(-\infty,\infty)$ As.: $x=0$

2. $f(x) = \log_{\frac{1}{2}} x$



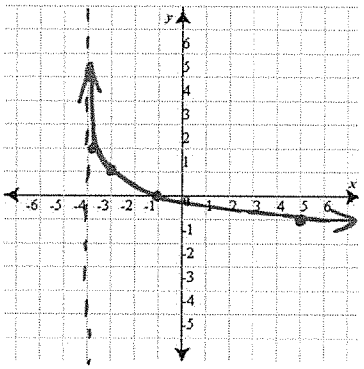
D: $(0,\infty)$ R: $(-\infty,\infty)$ As: $x=0$

3. $f(x) = 2\log_3 x$



D: $(0,\infty)$ R: $(-\infty,\infty)$ As: $x=0$

4. $f(x) = 1 + \log_{\frac{1}{3}}(x+4)$



D: $(-4,\infty)$ R: $(-\infty,\infty)$ As.: $x=-4$

Rules of Logs/Solving Equations with Logs

Condense each expression into one logarithm.

1. $\log 3 + \log(1+x)$

$\log(3(1+x))$

$= \log(3+3x)$

2. $4\log_\pi a - 3\log_\pi b$

$\log_\pi \frac{a^4}{b^3}$

Expand each expression.

3. $\log_3\left(\frac{6}{x}\right)$

$\log_3 6 - \log_3 x$

4. $\log(a^2b)^4$

$8\log a + 4\log b$

Solve each equation for x.

5. $1 = \log_5(2x+3)$

$5^1 = 2x+3$

$\boxed{x = 1}$

6. $\log_x 9 = \frac{2}{3}$

$x^{\frac{2}{3}} = 9$

$x = 9^{\frac{3}{2}}$

$x = (\sqrt{9})^3 = \boxed{27}$

7. $-\ln(4x) = \ln\left(\frac{1}{2}\right)$

$\ln\left(\frac{1}{4x}\right) = \ln\left(\frac{1}{2}\right)$

$\frac{1}{4x} = \frac{1}{2}$

$\boxed{x = \frac{1}{2}}$

8. $\log_2(x-1) + \log_2(x-3) = 3$

$\log_2(x^2-4x+3) = 3$

$x^2-4x+3 = 2^3$

$x^2-4x-5=0$

$(x-5)(x+1)=0$

$\boxed{x=5}$

~~$x=-1$~~

Counting Principle

1. You go to the cafeteria for lunch and have a choice of 4 entrees, 5 sides, 5 drinks, and 4 desserts. Assuming you have one of each category, how many different lunches could be made?

$4 \cdot 5 \cdot 5 \cdot 4 = \boxed{400}$

2. You go to the home electronics store to buy a new television. You have the following choices: lcd, dlp, crt, or plasma; full screen or wide screen; 13", 19" 27", 32", 36", 41", 51", or 63". How many different televisions does the store have to offer?

$4 \cdot 2 \cdot 8 = \boxed{64}$

3. You toss a penny 4 times. How many different outcomes are there?

$2 \cdot 2 \cdot 2 \cdot 2 = \boxed{16}$

Calculate your answers to the following two questions without using our formula for permutations.

4. How many different arrangements are there of the letters in the word FINALS?

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720}$

5. How many different arrangements are there of the letters in the word HAPPY?

Since two "P"s can be swapped → $\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \boxed{60}$

6. There are 8 students preparing to line up at the door – 4 girls and 4 boys. How many ways can they line up if the first in line must be a boy, the second must be a girl, and the last must be a boy?

$\underline{4}_{B} \cdot \underline{4}_{G} \cdot \underline{1} \cdot \underline{2} \cdot \underline{3} \cdot \underline{4} \cdot \underline{5} \cdot \underline{6}_{B} \rightarrow$ Front (Do these first) $= \boxed{5760}$

Permutations and Combinations

Calculate.

1. ${}_5C_2 = \frac{5!}{3!\cdot 2!} = \frac{5\cdot 4}{2} = 10$

2. ${}_4P_4 = \frac{4!}{0!} = 4\cdot 3\cdot 2\cdot 1 = 24$

3. ${}_4C_4 = \frac{4!}{4!\,0!} = 1$

4. ${}_{10}P_3 = \frac{10!}{7!} = 10\cdot 9\cdot 8 = 720$

5. ${}_{10}C_3 = \frac{10!}{7!\cdot 3!} = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} = 120$

6. ${}_3P_6 = 0$

Simplify.

12. $\frac{(n+1)!}{n^2+n} = \frac{(n+1)\cdot n\cdot (n-1)(n-2)\cdots 1}{(n+1)\cdot n} = (n-1)!$

13. $(n-2)!\,(n^2-n)$
$= (n-2)(n-1)\cdots 1\cdot (n-1)\cdot n$
$= n!$

14. a) At a math department meet and greet there are 12 new staff members. If everyone shakes everyone else's hand exactly once, how many handshakes will there be all together?

${}_{12}C_2 = \frac{12!}{10!\cdot 2!} = \frac{12\cdot 11}{2} = 66$

b) In how many different orders could the 12 new staff members be introduced?

${}_{12}P_{12} = \frac{12!}{0!} = 12\cdot 11\cdots 1 = 479{,}001{,}600$

Arithmetic/Geometric Sequences

Find the formula for the n^{th} term of the sequence, then use your formula to calculate the 10^{th} term.

1. 6, 15, 24, 33, ...

$a_1 = 6 \quad d = 9$

$a_n = 6 + 9(n-1)$

$a_{10} = 87$

2. 768, 384, 192, 96, ...

$a_1 = 768 \quad r = \frac{1}{2}$

$a_n = 768\left(\frac{1}{2}\right)^{n-1}$

$a_{10} = 1.5$

3. −5, −1, 3, 7, ...

$a_1 = -5 \quad d = 4$

$a_n = -5 + 4(n-1)$

$a_{10} = 31$

4. $\frac{2}{9}, \frac{2}{3}, 2, 6, \ldots$

$a_1 = \frac{2}{9} \quad r = 3$

$a_n = \frac{2}{9}(3)^{n-1}$

$a_{10} = 4374$

5. $a_2 = 5, \quad d = -1$

$a_1 = 6$

$a_n = 6 - 1(n-1)$

$a_{10} = -3$

6. $a_3 = 2, \quad a_5 = 6, \quad a_7 = 10$

$a_1 = -2 \quad d = 2$

$a_n = -2 + 2(n-1)$

$a_{10} = 16$

7. $a_1 = 3, \quad r = \frac{4}{3}$

$a_n = 3\left(\frac{4}{3}\right)^{n-1}$

$a_{10} = 39.95$

8. $a_2 = 4, \quad r = 2$

$a_1 = 2$

$a_n = 2(2)^{n-1}$

$a_{10} = 1024$

Arithmetic/Geometric Series

1. $\sum_{k=1}^{8} 3(2)^k = \frac{6(1-2^8)}{1-2}$

$= 1530$

2. $\sum_{k=2}^{24} (k+3) = \frac{23(5+27)}{2}$

$= 368$

3. $\sum_{k=1}^{30} 2k = \frac{30(2+60)}{2}$

$= 930$

4. $\sum_{k=-10}^{-1} \left(\frac{3}{2}k - 3\right) = \frac{10(-18-4.5)}{2}$

$= -112.5$

4. $\sum_{k=0}^{\infty} 7\left(\frac{1}{2}\right)^n = \frac{7(1-(\frac{1}{2})^{\infty})}{1-\frac{1}{2}}$

$= \frac{7}{1-\frac{1}{2}} = \frac{7}{\frac{1}{2}} = 14$

6. $\sum_{k=3}^{9} \frac{1}{8}(2)^3 = \frac{1(1-2^7)}{1-2}$

$= 127$

Random Series

1. $\sum_{k=1}^{13} (k^2 - 3k)$ Use formulas

$= \sum_{k=1}^{13} k^2 - 3\sum_{k=1}^{13} k$

$= 819 - 3(91) = 546$

2. $\sum_{k=-1}^{4} (k^3 - 3)$

$((-1)^3-3) + (0^3-3) + (1^3-3)$

$+ (2^3-3) + (3^3-3) + (4^3-3)$

$= 81$

3. $\sum_{k=1}^{4} \frac{2}{k} = \frac{2}{1} + \frac{2}{2} + \frac{2}{3} + \frac{2}{4}$

$= \boxed{4\frac{1}{6}}$

4. $\sum_{k=1}^{15} (k-6)(k+1)$

$= \sum_{k=1}^{15} (k^2 - 5k - 6)$

$= \sum_{k=1}^{15} k^2 - 5\sum_{k=1}^{15} k - 6\sum_{k=1}^{15} 1$

$= 1240 - 5(120) - 6(15)$

$= \boxed{550}$

Rewrite each sum using sigma notation.

5. $11 + 13 + 15 + 17 + 19$

$11+2(0)$ $11+2(1)$ $11+2(2)$ $11+2(3)$ $11+2(4)$

$\boxed{= \sum_{k=0}^{4} (11+2k)}$

6. $8 + 4 + 2 + 1 + 0.5$

2^3 2^2 2^1 2^0 2^{-1}

$\boxed{\sum_{k=-1}^{3} 2^k}$

7. $0 + 4 + 16 + 36 + 64 + 100 + 144$

0^2 2^2 4^2 6^2 8^2 10^2 12^2

$(2\cdot 0)^2$ $(2\cdot 1)^2$ $(2\cdot 2)^2$ $(2\cdot 3)^2$ $(2\cdot 4)^2$ $(2\cdot 5)^2$ $(2\cdot 6)^2$

$\boxed{\sum_{k=0}^{6} (2k)^2}$

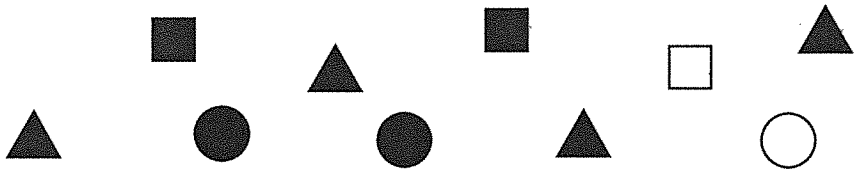
8. $1 - 3 + 9 - 27 + 81 - 243$

$(-3)^0$ $(-3)^1$ $(-3)^2$ $(-3)^3$ $(-3)^4$ $(-3)^5$

$\boxed{= \sum_{k=0}^{5} (-3)^k}$

Probability

For each question, you will be reaching into a bag and choosing randomly from the following selection of shapes. Calculate each probability and write your answer as a reduced fraction.



1. You select one shape at random. What is the probability that it is...

a) Not a circle?

$\frac{7}{10}$

b) Black or a square?

$\frac{9}{10}$

c) A triangle, circle, or square?

1

d) Black given that it is a triangle?

1

2. You select one shape at random, set it aside, then select a second shape at random. What is the probability that...

a) The first shape is black and the second shape is white?

$\frac{8}{10} \cdot \frac{2}{9} = \boxed{\frac{8}{45}}$

b) Both shapes are white circles?

$\frac{1}{10} \cdot \frac{0}{9} = \boxed{0}$

c) The first shape is a triangle and the second shape is black?

$\frac{4}{10} \cdot \frac{7}{9} = \boxed{\frac{14}{45}}$

d) Neither shape is a black?

$\frac{2}{10} \cdot \frac{1}{9} = \boxed{\frac{1}{45}}$

3. You select one shape at random, put it back into the bag, then select a second shape at random. What is the probability that...

a) The first shape is black and the second shape is white?

$\frac{8}{10} \cdot \frac{2}{10} = \boxed{\frac{4}{25}}$

b) Both shapes are white circles?

$\frac{1}{10} \cdot \frac{1}{10} = \boxed{\frac{1}{100}}$

c) The first shape is a triangle and the second shape is black?

$\frac{4}{10} \cdot \frac{8}{10} = \boxed{\frac{8}{25}}$

d) Neither shape is a black?

$\frac{2}{10} \cdot \frac{2}{10} = \boxed{\frac{1}{25}}$