

No calculator. Show all work for full credit. Simplify answers (including all roots) as much as possible.

1. Factor the polynomials completely.

a. $2x^2 - 32$

$$2(x^2 - 16)$$

$$\boxed{2(x+4)(x-4)}$$

b. $x^2 - x - 6$

$$\boxed{(x-3)(x+2)}$$

c. $mn^5 - m^3n$

$$mn(n^4 - m^2)$$

$$\boxed{mn(n^2 - m)(n^2 + m)}$$

d. $3x^2 + 5x + 2$

$$\boxed{(3x+2)(x+1)}$$

e. $x^3 - 3x^2 + x - 3$

$$x^2(x-3) + 1(x-3)$$

$$\boxed{(x-3)(x^2+1)}$$

f. $3x^3 - 6x^2 - 4x + 8$

$$3x^2(x-2) - 4(x-2)$$

$$\boxed{(x-2)(3x^2-4)}$$

2. Simplify the roots.

a. $\sqrt{64}$

$$\boxed{= 8}$$

b. $\sqrt{75}$

$$= \sqrt{25 \cdot 3}$$

$$\boxed{= 5\sqrt{3}}$$

c. $\sqrt{-81}$

$$\boxed{\text{undefined}}$$

d. $\sqrt{10000}$

$$\boxed{= 100}$$

e. $\sqrt{18}$

$$= \sqrt{9 \cdot 2}$$

$$\boxed{= 3\sqrt{2}}$$

f. $\sqrt{225}$

$$\boxed{= 15}$$

g. $\sqrt[3]{120}$

$$= \sqrt[3]{8 \cdot 15}$$

$$\boxed{= 2\sqrt[3]{15}}$$

h. $\sqrt[3]{-8}$ (careful!)

$$\boxed{= -2}$$

3. Solve the quadratic equation by factoring.

a. $x^2 + 10x + 16 = 0$

$$(x+8)(x+2) = 0$$

$$x+8=0 \quad x+2=0$$

$$\boxed{x = -8, -2}$$

c. $x^3 - 16x = 0$

$$x(x^2 - 16) = 0$$

$$x(x-4)(x+4) = 0$$

$$x=0 \quad x-4=0 \quad x+4=0$$

$$\boxed{x = 0, \pm 4}$$

4. Solve the quadratic equations by using square roots.

a. $x^2 - 150 = 0$

$$x^2 = 150$$

$$x = \pm \sqrt{150} = \pm \sqrt{25 \cdot 6}$$

$$\boxed{x = \pm 5\sqrt{6}}$$

c. $x^2 + 12 = 0$

$$x^2 = -12$$

$\boxed{\text{No solution}}$

e. $49x^2 - 1 = 0$

$$49x^2 = 1$$

$$x^2 = \frac{1}{49}$$

$$x = \pm \sqrt{\frac{1}{49}}$$

$$\boxed{x = \pm \frac{1}{7}}$$

b. $x^2 - 2x = 8$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x-4=0 \quad x+2=0$$

$$\boxed{x = 4, -2}$$

d. $x^3 - 2x^2 - 4x + 8 = 0$

$$x^2(x-2) - 4(x-2) = 0$$

$$(x-2)(x^2-4) = 0$$

$$(x-2)(x+2)(x-2) = 0$$

$$x-2=0 \quad x+2=0$$

$$\boxed{x = \pm 2}$$

b. $2x^2 = 90$

$$x^2 = 45$$

$$x = \pm \sqrt{45} = \pm \sqrt{9 \cdot 5}$$

$$\boxed{x = \pm 3\sqrt{5}}$$

d. $4x^2 - 16 = 0$

$$4x^2 = 16$$

$$x^2 = 4$$

$$x = \pm \sqrt{4}$$

$$\boxed{x = \pm 2}$$

f. $x^2 = 32$

$$x = \pm \sqrt{32} = \pm \sqrt{16 \cdot 2}$$

$$\boxed{x = \pm 4\sqrt{2}}$$

5. Solve these equations any way you want.

a. $-5x^2 + 15x = -20$

$$-5x^2 + 15x + 20 = 0$$

$$-5(x^2 - 3x - 4) = 0$$

$$-5(x-4)(x+1) = 0$$

$$x-4=0 \quad x+1=0$$

$$\boxed{x=4, -1}$$

b. $2x^5 - 2x^3 = 0$

$$2x^3(x^2 - 1) = 0$$

$$2x^3(x+1)(x-1) = 0$$

$$2x^3=0 \quad x+1=0 \quad x-1=0$$

$$\boxed{x=0, \pm 1}$$

6. The height of a diver above the water during a dive can be modeled by $h = -16t^2 + 32t + 48$, where h is the height in feet and t is the time in seconds. Find the time it takes for the diver to reach the water.

When it hits the water, its height is 0 so...

$$0 = -16t^2 + 32t + 48$$

$$0 = -16(t^2 - 2t - 3)$$

$$0 = -16(t-3)(t+1)$$

$$t-3=0 \quad t+1=0$$

$$t=3, -1$$

It didn't take negative time so answer is...

$$\boxed{3 \text{ seconds}}$$

7. Review:

a. Solve:

$$|3x-2| < 4$$

$$\begin{array}{ccc} -4 < 3x-2 < 4 \\ +2 & +2 & +2 \end{array}$$

$$\frac{-2}{3} < \frac{3x}{3} < \frac{6}{3}$$

$$\boxed{-\frac{2}{3} < x < 2}$$

b. Find the equation of the line perpendicular to $2x - y = 4$ going through the point $(2, -4)$

$$-y = -2x + 4$$

$$y = 2x - 4$$

$$\boxed{y+4 = -\frac{1}{2}(x-2)}$$

-or-

old slope is 2, so perpendicular slope is $-\frac{1}{2}$

$$\boxed{y = -\frac{1}{2}x - 3}$$

c. Solve the system of equations.

$$3x + y = 9 \rightarrow y = 9 - 3x$$

$$-2x - 5y = 7$$

$$-2x - 5(9 - 3x) = 7$$

$$-2x - 45 + 15x = 7$$

$$13x = 52$$

$$x = 4$$

$$\boxed{(4, -3)}$$

$$y = 9 - 3(4) = -3$$

d. Simplify. No negative exponents:

$$\frac{4x^{-3}}{x^6}$$

$$= \frac{4}{x^3 \cdot x^6} = \boxed{\frac{4}{x^9}}$$