Practice Test 4

Polynomials

For each polynomial state the degree, the leading coefficient, and the constant, and classify it as a monomial, binomial, or trinomial.

1. $7x - 2$

Degree: 1
Leading coefficient: 7
Constant: -2
Classification: binomial

2. $4x^5y^2z$

Degree: 8
Leading coefficient: 4
Constant: 0
Classification: monomial

3. 0.43

Degree: 0
Leading coefficient: 0.43
Constant: 0.43
Classification: monomial

4. $3a^2 + \sqrt{7}ab^4 - 2$

Degree: 5
Leading coefficient: $\sqrt{7}$
Constant: -2
Classification: trinomial

Not ok

5. Rewrite the expression $4x^{104} + 7x^{\sqrt{3}} - 2\sqrt{5}x^{-7} + \pi^2 x$ so that it is a polynomial. You are only allowed to change two of the numbers.

$4x^{104} + 7x^2 - 2\sqrt{5}x^3 + \pi x$

(Exponents have to be non-negative integers.)

Simplify.

6. $(3x^3 - 2) + (8 - 2x^3)$

$x^3 + 6$

7. $(xy - y^3) - (y^3 + xy^2 - xy)$

$2xy - 2y^3 - xy^2$

8. $(6k^3j^{-2})^0 - 4(9k)^0$

$1 - 4 = -3$

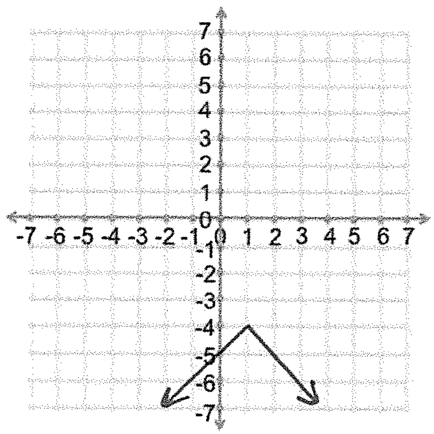
9. $7^{-1} + 2^{-2}$

$\frac{1}{7} + \frac{1}{4} = \frac{4}{28} + \frac{7}{28} = \frac{11}{28}$

10. Which number, -1, 5, $-\frac{1}{2}$, or $\frac{1}{5}$, could not possibly be a root of the polynomial $2x^3 - 7x^2 - 14x - 5$? Explain how you know.

$\frac{1}{5}$ could not be a root. According to the Rational Root Theorem, the denominator of the root must be a factor of the leading coefficient, but 5 is not a factor of 2.

11. Graph the function $f(x) = -|x - 1| - 4$ and state its domain and range.



Domain: $(-\infty, \infty)$

Range: $(-\infty, -4]$

Multiply and combine any like terms. Write your final answers without any negative exponents.

12. $(x - 5)(x + 1)$

$x^2 - 4x - 5$

13. $(3x + 5)^2 = (3x+5)(3x+5)$

$9x^2 + 30x + 25$

14. $3^{-2}(m^4 n^{-1})^{-2}$

$\frac{n^2}{9m^8}$

15. $(ab)^3(b - a^2 b)$

$a^3b^4 - a^5b^4$

16. $\frac{x^{-2}y^{-4}}{x^1 y^{-5}}$

$\frac{y}{x^3}$

17. $(3xy)^{-2}(6x^2 + 9y + 3xy)$

$\frac{2}{3y^2} + \frac{1}{x^2 y} + \frac{1}{3xy}$

18. A cylindrical can has a height of x centimeters. The radius of the base of the can is 5 centimeters shorter than its height. Find a formula for the volume of the can (*the volume of a cylinder is* $V = \pi r^2 h$). Write your final answer as a trinomial of degree 3.

$h = x,\ r = x-5$ so $V = \pi(x-5)^2 \cdot x = \pi(x^2-10x+25)x$

$V(x) = \pi x^3 - 10\pi x^2 + 25\pi x$

Factor out the GCF from each expression.

19. $12x^4 - 6x + 18x^3$

$6x(2x^3 - 1 + 3x^2)$

20. $2xy^3z^2 + x^2y^2z + 4y^2z$

$xy^2z(2yz + x + 4)$

Factor by grouping.

21. $(3ax + ax^2) + (9 + 3x)$

$= ax(3+x) + 3(3+x)$

$= (ax+3)(3+x)$

22. $4b - bc - 4ab + abc$

$= (4b - bc) - (4ab - abc)$

$= b(4-c) - ab(4-c)$

$= (b - ab)(4-c)$

$= b(1-a)(4-c)$

You can factor out the b here but you don't need to for full credit.

23. When a bullet is fired into the air at an angle of 45°, its height is given as a function of time by the equation $h(t) = -x^2 + 12x + 13$ where h is the height of the bullet in feet and t is the time in seconds since the gun was fired. How many seconds after the gun was fired will the bullet hit the ground?

The ground is a height of 0, so set

$0 = -x^2 + 12x + 13 = -(x^2 - 12x - 13) = -(x-13)(x+1)$

This equals 0 when $x = 13$ or -1. We can't have a negative amount of seconds, so the answer is 13 sec.

Factor.

24. $x^2 - 6x + 5$

$(x-5)(x-1)$

25. $b^2 + 2b + 1$

$(b+1)(b+1) = (b+1)^2$

either one

26. $z(2x+3)^2 + 3z(2x+3) + 2z$

$= z((2x+3)^2 + 3(2x+3) + 2)$

$= z((2x+3)+2)((2x+3)+1)$

$= z(2x+5)(2x+4)$

27. $x^2 - 9 = x^2 - 3^2$

$(x+3)(x-3)$

28. $(xy)^3 - 27$

$= (xy)^3 - 3^3$

$(xy-3)(x^2y^2 + 3xy + 9)$

29. $8y^9 + 1 = (2y^3)^3 + 1^3$

$(2y^3+1)(4y^6 - 2y^3 + 1)$

30. List five examples of rational numbers.

$0, -7, \frac{4}{9}, 0.281, -\frac{11}{3}$

31. List two examples of irrational numbers.

$\sqrt{11}, \pi$

32. a) True or false: All integers are rational numbers.

True.

b) Explain how you know your answer from part (a) is correct.

An integer, n, can be written as $\frac{n}{1}$ which is a fraction meaning n is rational.

Use the rational root theorem to list all possible rational roots for the following two polynomials. You do not have to test to see which numbers are actually roots.

33. $2x^5 - 7x^4 + 2x - 16$

$\pm \frac{1, 2, 4, 8, 16}{1, 2}$

$= \frac{1}{2}, -\frac{1}{2}, 1, -1, 2, -2, 4, -4, 8, -8, 16, -16$

34. $3x^{20} + x^{18} + 11$

$\pm \frac{1, 11}{1, 3}$

$= 1, -1, 11, -11, \frac{1}{3}, -\frac{1}{3}, \frac{11}{3}, -\frac{11}{3}$

35. According to the rational root theorem, the only possible rational roots for the polynomial $x^3 - 3x^2 - x + 3$ are 1, –1, 3, and –3. Which of these are actually roots for the polynomial, and which are not?

$1: (1)^3 - 3(1)^2 - (1) + 3 = 0$ ✓

$-1: (-1)^3 - 3(-1)^2 - (-1) + 3 = 0$ ✓

$3: (3)^3 - 3(3)^2 - (3) + 3 = 27 - 27 - 1 + 1 = 0$ ✓

$-3: (-3)^3 - 3(-3)^2 - (-3) + 3 = -27 - 27 + 3 + 3 = -48$ X

The roots are 1, -1, and 3

Divide.

36. $\dfrac{a^2+10a-24}{a+12} = a-2$

$$\begin{array}{r} a-2 \\ a+12\overline{)\,a^2+10a-24} \\ -\underline{\left(a^2+12a\right)} \\ -2a-24 \\ -\underline{\left(-2a-24\right)} \\ 0 \end{array}$$

(You could just realize here that you can factor the numerator into $(a+12)(a-2)$.)

37. $\dfrac{3x^3-14x^2+9x-4}{x-4} = 3x^2-2x+1$

$$\begin{array}{r} 3x^2-2x+1 \\ x-4\overline{)\,3x^3-14x^2+9x-4} \\ -\underline{\left(3x^3-12x^2\right)} \\ -2x^2+9x \\ -\underline{\left(-2x^2+8x\right)} \\ x-4 \\ -\underline{\left(x-4\right)} \\ 0 \end{array}$$

38. Factor the following polynomial completely (into the product of three binomials). (*Hint: Use the rational root theorem to find the first root and divide it out.*)

$$x^3+4x^2+5x+2$$

Possible roots are $1, -1, 2, -2$ (According to RRT)

Check the easiest: $(1)^3+4(1)^2+5(1)+2=12$ X

$(-1)^3+4(-1)^2+5(-1)+2=0$ ✓

Since -1 is a root we can divide out an $x+1$

$$\begin{array}{r} x^2+3x+2 \\ x+1\overline{)\,x^3+4x^2+5x+2} \\ -\underline{\left(x^3+x^2\right)} \\ 3x^2+5x \\ -\underline{\left(3x^2+3x\right)} \\ 2x+2 \\ -\underline{\left(2x+2\right)} \\ 0 \end{array}$$

So... $x^3+4x^2+5x+2=(x+1)(x^2+3x+2)$

$=(x+1)(x+1)(x+2)=(x+1)^2(x+2)$

Either one works as final answer.

Extra Credit:

Where will the function $f(x)=x^4-9x^2$ cross the x-axis?

A function crosses the x-axis when $y=0$. So set $x^4-9x^2=0$

Factor to get $x^2(x^2-9)$

$=x^2(x^2-3^2)=x^2(x+3)(x-3)$. This equals 0 when $x=0, 3,$ or -3, so we cross the x-axis at $(0,0)$, $(3,0)$ and $(-3,0)$.