Key

Practice Test 6

Graphing Rational Functions/Quadratics

Factor completely.

1. $9 - x^2y^2$

$(3+xy)(3-xy)$

2. $3x^2 + 30x + 75$

$3(x^2+10x+25) = 3(x+5)^2$

3. $a^3b^2c - a^2b^2c^2$

$a^2b^2c(a-c)$

4. $x^3 + 3x^2 - 4x - 12$

$x^2(x+3) - 4(x+3)$

$= (x+3)(x^2-4) = (x+3)(x+2)(x-2)$

For each function, find the domain and vertical and horizontal asymptotes (if any). For the asymptotes, your answer must be the equation of the line, not just a number.

5. $f(x) = \frac{1}{x^2}$

Domain: $\{x \in \mathbb{R} \mid x \neq 0\}$

Vertical Asymptote: $x = 0$

Horizontal Asymptote: $y = 0$

6. $f(x) = \frac{-1}{x+4} + 3$

Domain: $\{x \in \mathbb{R} \mid x \neq -4\}$

Vertical Asymptote: $x = -4$

Horizontal Asymptote: $y = 3$

7. $f(x) = \frac{2x^2+8x+6}{x^2+2x-3} = \frac{2\cancel{(x+3)}(x+1)}{\cancel{(x+3)}(x-1)}$

Domain: $\{x \in \mathbb{R} \mid x \neq -3, 1\}$

Vertical Asymptote: $x = 1$

Horizontal Asymptote: $y = 2$

8. $f(x) = 1 + \frac{x+1}{x-2}$

Domain: $\{x \in \mathbb{R} \mid x \neq 2\}$

Vertical Asymptote: $x = 2$

Horizontal Asymptote: $y = 2$

Fill in the blank to make each trinomial a perfect square and factor.

9. $x^2 + 2x + \underline{1} = (x+1)^2$

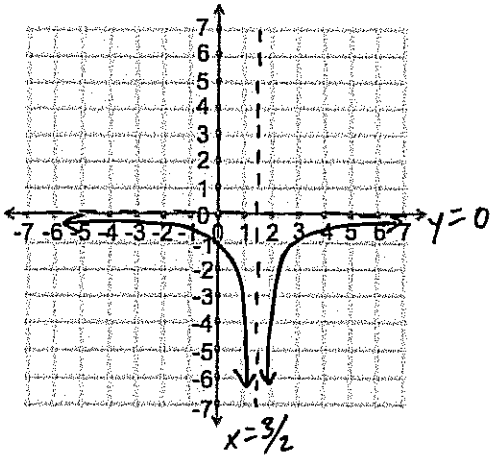
10. $x^2 - \frac{2}{3}x + \underline{1/9} = (x - 1/3)^2$

11. Let $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials. If n is a root of Q, and f does not have a vertical asymptote at $x = n$, what must be true of $P(n)$? Explain.

$P(n) = 0$. Since $x = n$ is not an asymptote, the factor $x-n$ in the denominator must cancel with an $x-n$ in the numerator. This means n is also a root of P.

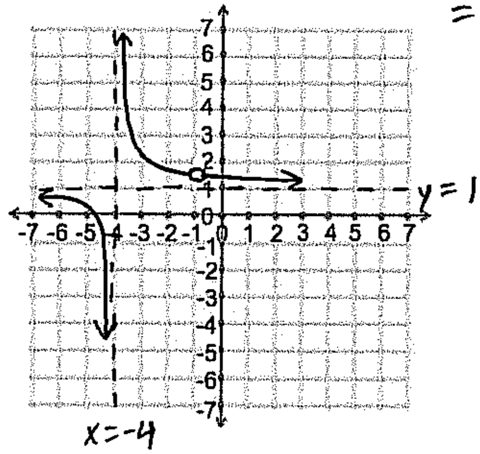
Graph each function. If there are any asymptotes, label them.

12. $f(x) = \frac{-3}{(2x-3)^2}$

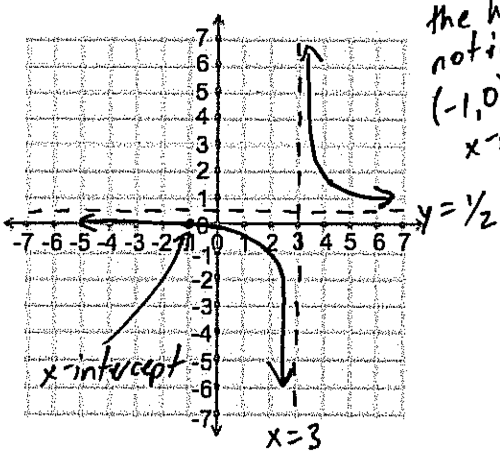


13. $f(x) = \frac{x+1}{x^2+5x+4} + 1$ $= \frac{\cancel{x+1}}{\cancel{(x+1)}(x+4)} + 1$

$= \frac{1}{x+4} + 1$

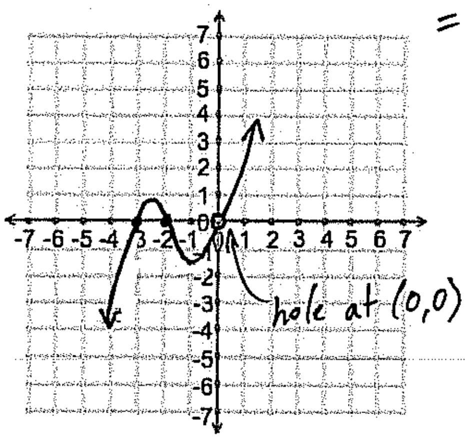


14. $f(x) = \frac{x+1}{2x-6}$ $= \frac{x+1}{2(x-3)}$ To help find the location of the hyperbolas, notice that (-1,0) is the x-intercept.



15. $f(x) = \frac{x^4+5x^3+6x^2}{x}$ $= \frac{x^{\cancel{2}}(x+2)(x+3)}{\cancel{x}}$

$= x(x+2)(x+3)$



16. a) List all possible rational roots of the polynomial $x^4 + x^3 - 3x^2 - x + 2$ according to the rational root theorem.

$\frac{\pm 1, 2}{1} = \pm 1, \pm 2$

b) Find which of your answers to the previous question are actually roots of the polynomial.

$1: 1^4 + 1^3 - 3(1)^2 - 1 + 2 = 0$ ✓

$-1: (-1)^4 + (-1)^3 - 3(-1)^2 - (-1) + 2 = 0$ ✓

$2: 2^4 + 2^3 - 3(2)^2 - 2 + 2 = 12$ X

$-2: (-2)^4 + (-2)^3 - 3(-2)^2 - (-2) + 2 = 0$ ✓

1, -1, and -2 are roots. 2 is not.

Solve for x by completing the square.

17. $x^2 - 2x - 5 = 0$

$x^2 - 2x = 5$

$x^2 - 2x + 1 = 6$

$(x-1)^2 = 6$

$x - 1 = \pm 2.45$

$x = 3.45, -1.45$

18. $x^2 = -9 - 10x$

$x^2 + 10x = -9$

$x^2 + 10x + 25 = 16$

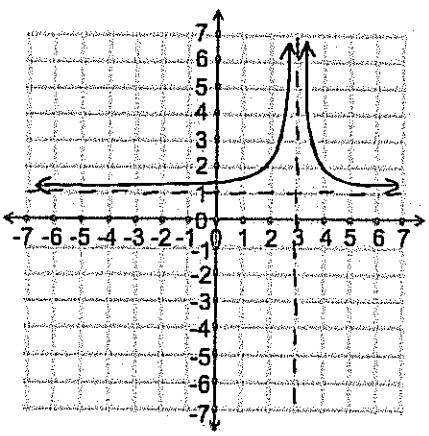
$(x+5)^2 = 16$

$x + 5 = \pm 4$

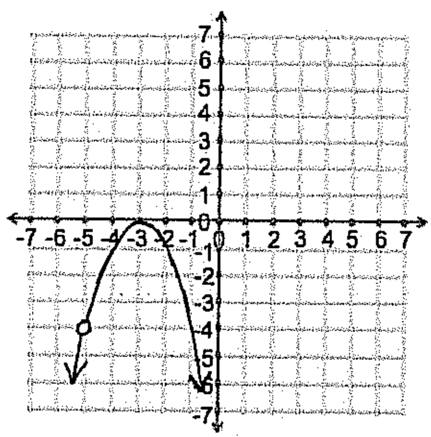
$x = -1, -9$

What functions are given in each graph?

19. $f(x) = \frac{1}{(x-3)^2} + 1$



20. $f(x) = \frac{-(x+3)^2(x+5)}{x+5}$



Extra Credit. Create a function that has a domain of $\{x \in \mathbb{R} | x \neq -2, 0\}$, a vertical asymptote of $x = -2$, a horizontal asymptote of $y = 3$, and an x-intercept of $(-1,0)$. Then graph your function.

$$f(x) = \frac{3x(x+1)}{x(x+2)}$$