Key

Properties of Exponents

Simplify.

1. $(3a^{-1}b^2)^2$

$= \frac{9b^4}{a^2}$

2. $\left(\frac{x^0 y}{2}\right)^{-1}$

$= \frac{2}{y}$

3. $\frac{x^{-3}y^4}{x^{-4}y^{-5}} = xy^9$

4. $(2xy)^{-1}(2x + 2y + xy)$

$= \frac{1}{y} + \frac{1}{x} + \frac{1}{2}$

5. $(3ab^2)(2ab)^2(5a)$

$60a^5b^4$

6. $2^{-2} + 3^{-2}$

$\frac{1}{4} + \frac{1}{9} = \frac{13}{36}$

Classifying Polynomials

For each polynomial state its degree classification (constant, linear, etc.), the leading coefficient, and the constant, and classify it as a monomial, binomial, or trinomial.

7. $3 - x$

Degree Classification: linear
Leading coefficient: -1
Constant: 3
Classification: binomial

8. $2x^4$

Degree Classification: quartic
Leading coefficient: 2
Constant: 0
Classification: monomial

9. -1

Degree Classification: constant
Leading coefficient: -1
Constant: -1
Classification: monomial

10. $-z + 5z^2 + 1$

Degree Classification: quadratic
Leading coefficient: 5
Constant: 1
Classification: trinomial

11. $x^3 + 16x$

Degree Classification: cubic
Leading coefficient: 1
Constant: 0
Classification: binomial

12. $-3 + \frac{x^4}{2}$

Degree Classification: quartic
Leading coefficient: ½
Constant: -3
Classification: binomial

Factoring

Factor Completely.

13. $x^2 + x - 6$

$= (x+3)(x-2)$

14. $9b^2 - 4$

$= (3b+2)(3b-2)$

15. $2x^2y + 4xy + 2y$

$= 2y(x^2+2x+1)$

$= 2y(x+1)^2$

16. $z^3 + 8$

$= (z+2)(z^2-2z+4)$

17. $n^9 - 1$

$= (n^3-1)(n^6+n^3+1)$

$= (n-1)(n^2+n+1)(n^6+n^3+1)$

18. $16 - 2x^3$

$= 2(8-x^3)$

$= 2(2-x)(4+2x+x^2)$

19. $x^3 + x^2 - x - 1$

$x^2(x+1) - 1(x+1)$

$= (x^2-1)(x+1)$

$= (x-1)(x+1)^2$

20. $54 + 2a^3$

$2(27+a^3)$

$= 2(3+a)(9-3a+a^2)$

Rational Root Theorem

Use the rational root theorem to list all possible rational roots for the following two polynomials. You do <u>not</u> have to test to see which numbers are actually roots.

21. $4x^3 - x^2 + 6x + 3$

$\frac{\pm 1, 3}{1, 2, 4} = \pm 1, 3, \frac{1}{2}, \frac{3}{2}, \frac{1}{4}, \frac{3}{4}$

22. $12x^5 - 4x + 1$

$\frac{\pm 1}{1, 2, 3, 4, 6, 12} = \pm 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}$

23. $10x^{12} + 7$

$\frac{\pm 1, 7}{1, 2, 5, 10}$

$= \pm 1, \frac{1}{2}, \frac{1}{5}, \frac{1}{10}, 7, \frac{7}{2}, \frac{7}{5}, \frac{7}{10}$

24. $2x^2 + x + 2$

$\pm \frac{1, 2}{1, 2} = \pm 1, 2, \frac{1}{2}$

25. According to the rational root theorem, the only possible rational roots for the polynomial $-x^4 - 2x^2 + 3$ are 1, –1, 3, and –3. Which of these are actually roots for the polynomial, and which are not?

$1: -(1)^4 - 2(1)^2 + 3 = 0$ ✓

$-1: -(-1)^4 - 2(-1)^2 + 3 = 0$ ✓

$3: -(3)^4 - 2(3)^2 + 3 = -96$ X

$-3: -(-3)^4 - 2(-3)^2 + 3 = -96$ X

$\boxed{1 + -1 \text{ are roots}}$

26. Find all of the rational roots to the polynomial $4x^2 - 1$ by using the rational root theorem to find all possibilities then testing them.

possible roots: $\pm \frac{1}{1,2,4} = \pm \frac{1}{2}, 1, \frac{1}{4}$

$1: 4(1)^2 - 1 = 3$ X

$\frac{1}{2}: 4(\frac{1}{2})^2 - 1 = 0$ ✓

$\frac{1}{4}: 4(\frac{1}{4})^2 - 1 = -\frac{3}{4}$ X

$\frac{1}{2}$ + $-\frac{1}{2}$ are the only rational roots. (Don't need to check negatives since a negative^2 = positive^2.)

Dividing Polynomials

27. $\frac{x^2+4x+3}{x+1} = x + 3$

$$\begin{array}{r}
x + 3 \\
x+1 \overline{\smash{)}\, x^2 + 4x + 3} \\
-\underline{(x^2 + x)} \\
3x + 3 \\
-\underline{(3x + 3)} \\
0
\end{array}$$

28. $\frac{2x^2+7x+5}{2x+5} = x + 1$

$$\begin{array}{r}
x + 1 \\
2x+5 \overline{\smash{)}\, 2x^2 + 7x + 5} \\
-\underline{(2x^2 + 5x)} \\
2x + 5 \\
-\underline{(2x + 5)} \\
0
\end{array}$$

29. $(x^3 + 5x^2 + 5x - 2) \div (x + 2)$

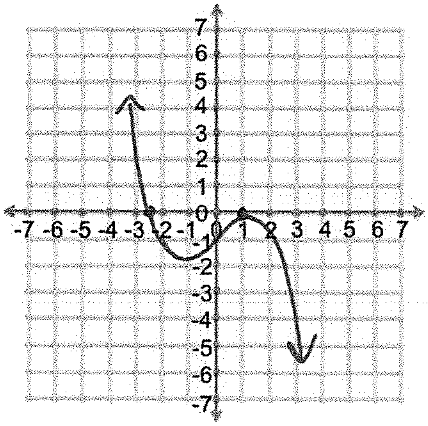
$$\begin{array}{r}
x^2 + 3x - 1 \\
x+2 \overline{\smash{)}\, x^3 + 5x^2 + 5x - 2} \\
-\underline{(x^3 + 2x^2)} \\
3x^2 + 5x \\
-\underline{(3x^2 + 6x)} \\
-x - 2 \\
-\underline{(-x - 2)} \\
0
\end{array}$$

30. $\frac{x^3-27}{x^2+3x+9} = x - 3$

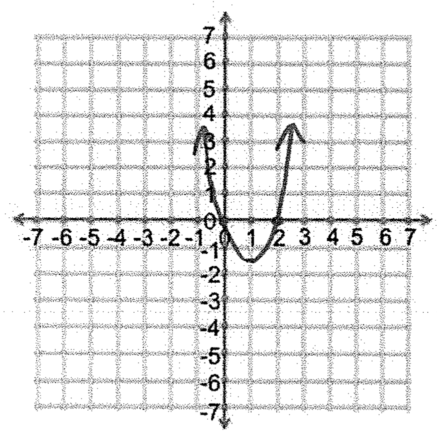
$$\begin{array}{r}
x - 3 \\
x^2+3x+9 \overline{\smash{)}\, x^3 + 0x^2 + 0x - 27} \\
-\underline{(x^3 + 3x^2 + 9x)} \\
-3x^2 - 9x - 27 \\
-\underline{(-3x^2 - 9x - 27)} \\
0
\end{array}$$

Graphing Polynomials

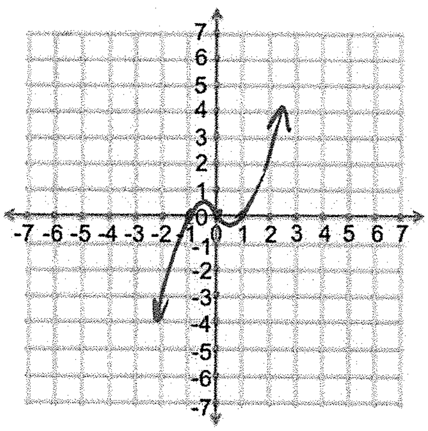
31. $f(x) = -(x-1)^2(2x+5)$



32. $f(x) = x^2 - 2x = x(x-2)$

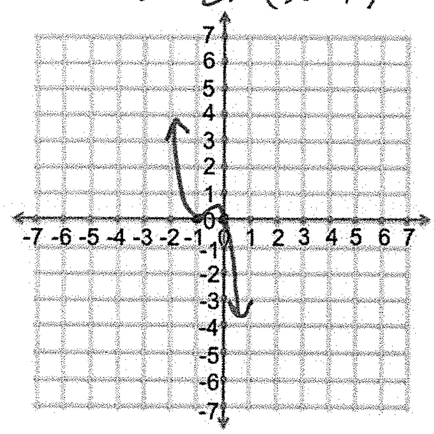


33. $f(x) = x^3 - x = x(x+1)(x-1)$



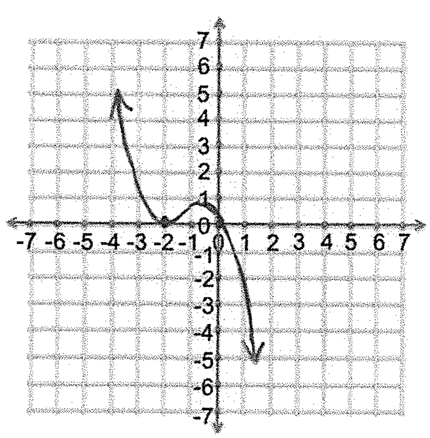
34. $f(x) = -2x^3 - 4x^2 - 2x$

$= -2x(x+1)^2$



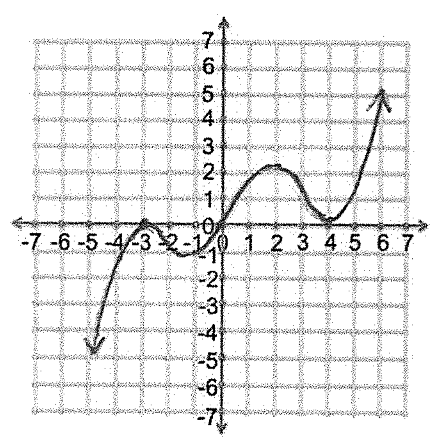
What polynomials are graphed below?

35.



$f(x) = -x(x+2)^2$

36.



$f(x) = x(x-4)^2(x+3)^2$

Solving Equations with Rational Expressions

37. $\left(\frac{1}{2}+\frac{1}{x}=\frac{5}{6}\right)6x$

$3x+6=5x$

$6=2x$

$x=3$

38. $\left(\frac{1}{x}+\frac{1}{x-1}=\frac{3}{2}\right)2x(x-1)$

$2(x-1)+2x=3x(x-1)$

$4x-2=3x^2-3x$

$3x^2-7x+2=0$

$(3x-1)(x-2)=0$

$x=1/3, 2$

39. $\left(\frac{4}{3x^2}-\frac{1}{3}=\frac{1}{x}\right)3x^2$

$4-x^2=3x$

$x^2+3x-4=0$

$(x+4)(x-1)=0 \quad x=-4,1$

40. $\left(\frac{2}{x-2}+\frac{1}{x^2-4}=\frac{1}{x+2}\right)(x+2)(x-2)$

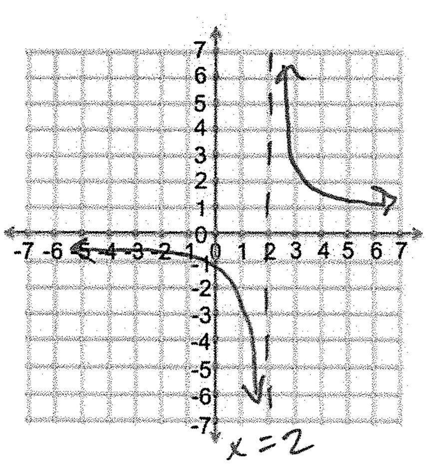
$2(x+2)+1=x-2$

$x=-7$

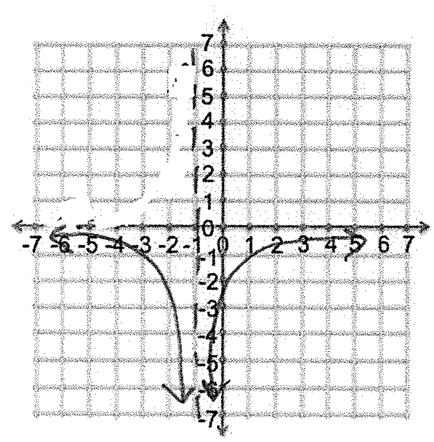
Finding vertical asymptotes

Graph each function. Label the vertical asymptotes.

41. $f(x)=\frac{1}{x-2}$



42. $f(x)=\frac{-1}{(x+1)^2}$



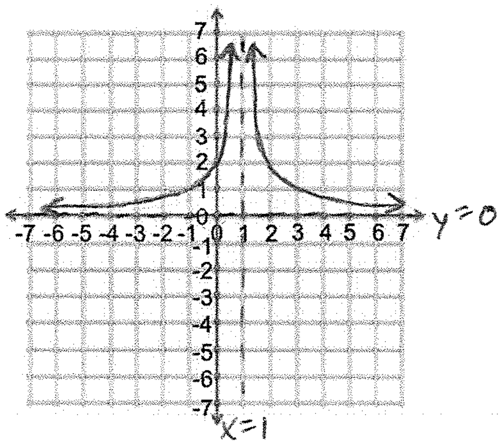
43. Give two different functions: one with a vertical asymptote at $x = 1$, and the other with a hole at $x = 1$.

$f(x)=\frac{1}{x-1}$ asymptote

$g(x)=\frac{x-1}{x-1}$ hole

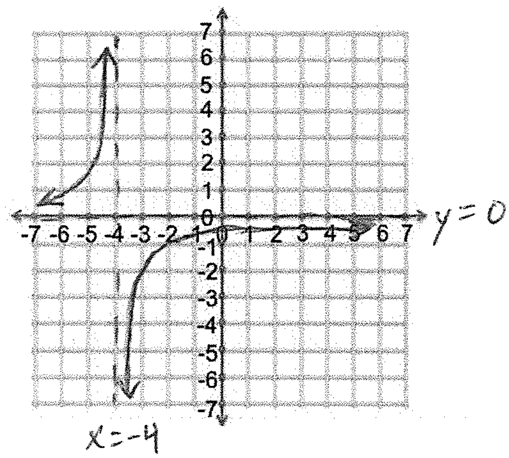
What rational functions are graphed below?

44.



$f(x) = \frac{1}{(x-1)^2}$

45.



$f(x) = \frac{-1}{x+4}$

Finding horizontal asymptotes

46. Let $f(x) = \frac{P(x)}{Q(x)}$ be a rational function. How can the horizontal asymptote (if there is one) be found in the following three cases?

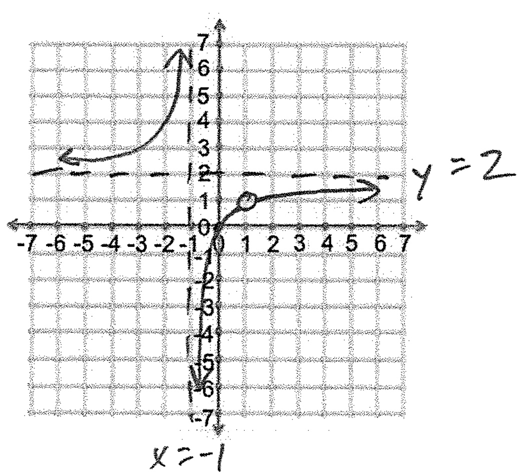
a) degree(P) > degree(Q) No horizontal asymptote

b) degree(P) < degree(Q) $y = 0$

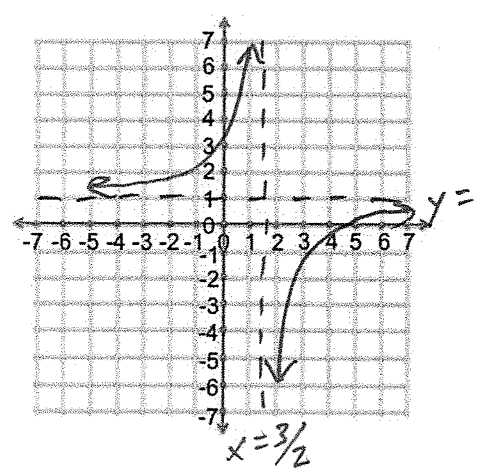
c) degree(P) = degree(Q) $y = \frac{\text{leading coefficient of } P}{\text{leading coefficient of } Q}$

Graph each rational function. Label vertical and horizontal asymptotes.

47. $f(x) = \frac{2x^2-2x}{x^2-1} = \frac{2x\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$



48. $f(x) = \frac{-1}{2x-3} + 1$



49. True or false: All polynomials are rational functions. If true, explain why. If false, give an example that shows why it is false.

True... you can write a polynomial, $P(x)$, as $\frac{P(x)}{1}$ and 1 is a polynomial (a constant).

50. True or false: All rational functions are polynomials. If true, explain why. If false, give an example that shows why it is false.

False... $\frac{x+1}{x^2-2}$ is not a polynomial.

Completing the square

51. Let $P(x)$ be a quadratic trinomial with a leading coefficient of 1. What must be true about the linear coefficient and the constant in order for $P(x)$ to factor into a perfect square?

Solve by completing the square.

52. $x^2 - 2x - 8 = 0$

$x^2 - 2x + 1 - 1 - 8 = 0$

$(x-1)^2 - 9 = 0$

$x - 1 = \pm 3$

$x = 4, -2$

53. $x^2 + 4x + 2 = 0$

$x^2 + 4x + 4 - 4 + 2 = 0$

$(x+2)^2 = 2$

$x + 2 = \pm 1.4$

$x = -0.6, -3.4$

54. $3x^2 + 6x - 1 = 0$

$3(x^2 + 2x + 1 - 1) - 1 = 0$

$3(x+1)^2 = 4$

$(x+1)^2 = 4/3$

$x + 1 = \pm 1.5$

$x = 0.5, -2.5$

55. $-x^2 + 4x + 6 = 0$

$-(x^2 - 4x + 4 - 4) + 6 = 0$

$-(x-2)^2 = -10$

$x - 2 = \pm 3.2$

$x = 5.2, -1.2$

Quadratic Formula

Solve with the quadratic formula.

56. $x^2 - 2x - 8 = 0$

$$x = \frac{2 \pm \sqrt{4-4(-8)}}{2}$$

$$x = \frac{2 \pm 6}{2} = 4, -2$$

57. $x^2 + 4x + 2 = 0$

$$x = \frac{-4 \pm \sqrt{16-4(2)}}{2}$$

$$= \frac{-4 \pm 2.8}{2} = -0.6, -3.4$$

58. $x^2 = 2x + 1$

$$x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{4-4(-1)}}{2}$$

$$= \frac{2 \pm 2.8}{2} = -0.4, 2.4$$

59. $x^2 + 3x + 7 = 1 - x + 2x^2$

$$x^2 - 4x - 6 = 0$$

$$x = \frac{4 \pm \sqrt{16-4(-6)}}{2}$$

$$= \frac{4 \pm 6.3}{2} = 5.15, -1.15$$

Vertex form of a parabola

Put each parabola in vertex form and state the vertex.

60. $f(x) = x^2 + 2x + 5$

$$= x^2 + 2x + 1 - 1 + 5$$

$$= (x+1)^2 + 4$$

Vertex: $(-1, 4)$

61. $f(x) = 2x^2 - 5x - 2$

$$= 2(x^2 - 2.5x) - 2$$

$$= 2(x^2 - 2.5x + 1.56 - 1.56) - 2$$

$$= 2(x - 1.25)^2 - 5.125$$

Vertex: $(1.25, -5.125)$

Find the vertex of each parabola by using vertex = $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

62. $f(x) = x^2 + 2x - 1$

$$-\frac{b}{2a} = \frac{-2}{2} = -1$$

$$f(-1) = (-1)^2 + 2(-1) - 1 = -2$$

Vertex: $(-1, -2)$

63. $f(x) = 3x^2 - 3x + 7$

$$-\frac{b}{2a} = \frac{3}{2(3)} = \frac{1}{2}$$

$$f(1/2) = 3(1/2)^2 - 3(1/2) + 7 = 6\,1/4$$

Vertex: $(0.5, 6.25)$