

## Course of study - Geometry

1. **Introduction:** Geometry is a course offered in the Mathematics Department. The course will be primarily taken by students in Grades 9 and 10, but the course may include 11<sup>th</sup> and 12 graders as well. The course is a two semester course.

### 2. Course Description

- Geometry is a one year geometry course which covers the state of California Common Core Math State Standards (CaCCSS-M) for geometry. Over the course of the year, students will participate in a broad range of instructional experiences using a variety of methods including the Common Core mathematical practices. The essential program goals are finding surface area and volume, understanding relationships between lines and angles, using deductive reasoning to communicate logical arguments and formulate proofs, identifying and justifying congruence of objects and/or their parts, and using proportional reasoning to identify and justify similarity between objects including right triangles and trigonometry. Successful completion of this course will prepare students for the second year of algebra and precalculus. The intended audience is for all students who have the prerequisite skills of Common Core Algebra Program Goals.

The Common Core Mathematical Practices are:

*Make sense of problems and persevere in solving them.*

*Reason abstractly and quantitatively.*

*Construct viable arguments and critique the reasoning of others.*

*Model with mathematics.*

*Use appropriate tools strategically.*

*Attend to precision.*

*Look for and make use of structure.*

*Look for and express regularity in repeated reasoning.*

### 3. Prerequisite skills and knowledge suggested for success in the course:

- Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.
- Identify and use radicals including simplifying, rationalizing, adding, subtracting, multiplying, dividing, and estimating decimal approximations.
- Solve multi-step equations and word problems. Students should be able to check the solution for accuracy and reasonableness.
- Graph lines, and locate points on a coordinate plane.
- Analyze problems by identifying relationships, distinguishing relevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

Justification: Geometry is a challenging course that requires students to have a solid foundation in number sense and solving equations. Students must also understand the x-y coordinate system, how to plot points, and graph lines. These prerequisite skills will allow the focus of the course to be on the course goals.

### 4. List of program goals/learning outcomes to be met.

- **Goal-1:** Students will make valid conclusions about relationships between lines and angles
- **Goal-2:** Students will use deductive reasoning to communicate logical arguments and formulate proofs.
- **Goal-3:** Students will identify and justify the congruence of objects and/or their parts.
- **Goal-4:** Students will use proportional reasoning to identify and justify similarity between objects including right triangles and trigonometry.
- **Goal-5:** Students will analyze and solve problems using the surface area of two and three dimensional figures, and the volume of three dimensional objects.

**5. Learning progressions and proficiency scales** aligned with the program goals listed above

Goal 1: Students will make valid conclusions about relationships between lines and angles.

<p>Advanced Content (4.0): What is the more complex use of content knowledge or skill required of students to master this goal?</p>	<p>Students will be able to solve for missing angles/segment lengths when provided with minimal information with more elaborate diagrams. More advanced algebra will be used.</p>
<p>Target Content (3.0): What is the specific target content knowledge or skill required of students to progress towards master of this goal?</p>	<p>Students will be able to write algebraic equations involving angle and segment relationships.</p> <p>(Example A-Solve for x:</p> <div style="text-align: center;">  </div> <p>Example B-Three times the complement of an angle is 14° more than the supplement.)</p> <p>Students will be able to find the sum of the interior and exterior angle measures for polygons.</p>
<p>Emerging Content (2.0): What basic terminology, specific facts, or simple ideas are required for students to progress toward mastery of this goal?</p>	<p>Students will be able to identify, visualize, and sketch the following objects and know their relationships: parallel, perpendicular, and skew lines, complementary, supplementary, vertical, and linear pair angles, angle bisectors, perpendicular bisectors. (ie: Students can sketch a linear pair and know the angles are supplementary.)</p> <p>Students will be able to determine missing angle measures based on given information.</p> <p>Students will be able to use the triangle sum theorem.</p>
<p>Basic Content (1.0): Students understand only basic knowledge of the content.</p>	
<p>Below Basic Content (0.0): Even with help, student has no understanding of the concept. Student demonstrates need for additional support in gaining the basic or prerequisite skill.</p>	

Prerequisite skills for line and angle relationships:

- Solving equations.
- Graphing lines and points on the coordinate plane, including identifying parallel and perpendicular lines

Goal 2: Students will use deductive reasoning to communicate logical arguments and formulate proof.

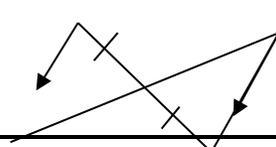
<p>Advanced Content (4.0): What is the more complex use of content knowledge or skill required of students to master this goal?</p>	<p>Students will be able to design and write a complete proof. This may prove a theorem, or a conjecture. Students should be competent in all forms of proofs including two-column, narrative, flow, symbolic or proof by contradiction.</p> <p>Students will be able to construct viable arguments and critique the reasoning of other arguments.</p> <p>Given a situation, students will be able to both select and use the appropriate type of reasoning to justify their conclusion.</p>
<p>Target Content (3.0): What is the specific target content knowledge or skill required of students</p>	<p>Given a picture, the given and prove, students will be able to write persuasive and valid proof (Students may choose two column,</p>

to progress towards master of this goal?	<p>flowchart or narrative/paragraph proof).</p> <p>Students will be able to identify one or more mistakes in a presented proof.</p> <p>Students will be able to justify their conclusions using postulates, theorems, properties or definitions.</p>
Emerging Content (2.0): What basic terminology, specific facts, or simple ideas are required for students to progress toward mastery of this goal?	<p>Students will be able to fill in missing statements or reasons to complete a proof (either flow or two-column format).</p> <p>Students will be able to recognize the structure of proof formats, identifying critical components of the proof (given hypothesis, conclusion, statements, reasons, etc.)</p> <p>Given all statements and reasons to a proof without order, students will be able to place them in a logical sequence.</p> <p>Students will be able to judge the validity of a simple logical argument or logic chain (converse isn't always true). If not true, be able to provide a counterexample.</p>
Basic Content (1.0): Students understand only basic knowledge of the content.	
Below Basic Content (0.0): Even with help, student has no understanding of the concept. Student demonstrates need for additional support in gaining the basic or prerequisite skill.	

Prerequisite skills for proof writing:

- Conditional Statements (conditional, converse, inverse and contrapositive)
- Valid Conclusions
- Contradictions and counter-examples
- Logic chains
- Euler Diagrams
- Familiarity with proof types
- Properties of equality / congruence
- Application of postulates, properties, theorems, and definitions and their roles in proof and conclusions.
- Differentiation between inductive and deductive reasoning

Goal 3: Students will have the ability to identify and justify congruence of objects and/or their parts.

Advanced Content (4.0): What is the more complex use of content knowledge or skill required of students to master this goal?	Students will be able to prove congruent triangles in complex diagrams (i.e. multiple triangles, distracting information, etc)
Target Content (3.0): What is the specific target content knowledge or skill required of students to progress towards master of this goal?	<p>Students will be able to:</p> <ul style="list-style-type: none"> <li>• Use congruency postulates to: <ul style="list-style-type: none"> <li>○ Identify congruent triangles <ul style="list-style-type: none"> <li>▪ For triangles, use SSS, SAS, ASA, AAS, &amp; H-L</li> <li>▪ Understand why ASS and AAA do not create rigid triangles</li> <li>▪ Determine if the following triangles are congruent given a diagram and/or limited information. For example,</li> </ul> </li> </ul> </li> </ul> 

	<ul style="list-style-type: none"> <li>○ Find missing sides and angles given a congruence statement</li> <li>● Prove and apply the Isosceles Triangle Theorem (and its converse)</li> <li>● Prove two triangles are congruent using SSS, SAS, ASA, AAS</li> </ul>
Emerging Content (2.0): What basic terminology, specific facts, or simple ideas are required for students to progress toward mastery of this goal?	<p>Students will be able to...</p> <ul style="list-style-type: none"> <li>● Understand the basic concept of congruence <ul style="list-style-type: none"> <li>○ Definition</li> <li>○ Identify corresponding sides and angles of polygons</li> </ul> </li> <li>● Write a congruency statement <ul style="list-style-type: none"> <li>○ List corresponding angles and sides in proper sequence</li> <li>○ Corresponding Parts of Congruent Triangles are Congruent (CPCTC)</li> </ul> </li> </ul>
Basic Content (1.0): Students understand only basic knowledge of the content.	
Below Basic Content (0.0): Even with help, student has no understanding of the concept. Student demonstrates need for additional support in gaining the basic or prerequisite skill.	

Prerequisite skills for congruence:

- Definition of congruence
- Introduction to rigid motions (transformations: translations, reflections, rotations)
- Identifying corresponding sides and angles in two polygons
- Identify the legs and hypotenuse of a right triangle
- Identify the base angles, vertex angle, legs, and base of an isosceles triangle

Goal 4: Students will use proportional reasoning to identify and justify similarity between objects including right triangles and trigonometry.

Advanced Content (4.0): What is the more complex use of content knowledge or skill required of students to master this goal?	<p>Students will be able to...</p> <ul style="list-style-type: none"> <li>● Prove similar polygons using similarity postulates</li> <li>● Apply trigonometric ratios to real-world modeling</li> </ul>
Target Content (3.0): What is the specific target content knowledge or skill required of students to progress towards master of this goal?	<p>Students will be able to...</p> <ul style="list-style-type: none"> <li>● Understand what sine, cosine, and tangent represent conceptually (i.e. all triangles with congruent angles have similar side length ratios)</li> <li>● Use similarity postulates to: <ul style="list-style-type: none"> <li>○ Identify similar polygons <ul style="list-style-type: none"> <li>▪ For triangles, use SSS, SAS, AA, H-L</li> </ul> </li> <li>○ Find missing sides and angles <ul style="list-style-type: none"> <li>▪ Demonstrate similarity using proportions</li> </ul> </li> </ul> </li> <li>● Find missing sides and angles within a right triangle using: <ul style="list-style-type: none"> <li>○ Trigonometric ratios (sine, cosine, tangent)</li> <li>○ Special Right Triangle ratios (30-60-90, 45-45-90)</li> </ul> </li> </ul>

Emerging Content (2.0): What basic terminology, specific facts, or simple ideas are required for students to progress toward mastery of this goal?	Students will be able to... <ul style="list-style-type: none"> <li>• Identify opposite, adjacent, and hypotenuse sides of a right triangle</li> <li>• Define trigonometric ratios (SOHCAHTOA)</li> <li>• Understand the basic concept of similarity <ul style="list-style-type: none"> <li>○ Definition</li> <li>○ Identify corresponding sides of polygons</li> <li>○ Write corresponding side length proportions</li> </ul> </li> </ul>
Basic Content (1.0): Students understand only basic knowledge of the content.	
Below Basic Content (0.0): Even with help, student has no understanding of the concept. Student demonstrates need for additional support in gaining the basic or prerequisite skill.	

Prerequisite skills for similarity:

- Algebraic skills include:
  - Solving equations with proportions (cross-multiplying)
  - Operations and simplification of radicals (rationalizing the denominator)
  - Pythagorean Theorem
- Formulate a proof in various formats

Goal 5 : Students will analyze and solve problems using the surface area of two and three dimensional figures, and the volume of three dimensional objects.

Advanced Content (4.0): What is the more complex use of content knowledge or skill required of students to master this goal?	Students will be able to calculate surface area and volume using non-routine shapes and values (i.e.: 20-gon, oblique, icosahedron) and non-routine modeling situations involving cost, comparisons or combinations of solids, quantity of materials, etc.
Target Content (3.0): What is the specific target content knowledge or skill required of students to progress towards master of this goal?	Students will be able to use the formulas to calculate surface area and volume of solids. Some dimensions must be calculated using right triangles, trigonometry, and/or other provided information such as circumference.
Emerging Content (2.0): What basic terminology, specific facts, or simple ideas are required for students to progress toward mastery of this goal?	Students will be able to use a given formula to calculate the surface area and volume of a shape with given dimensions.
Basic Content (1.0): Students understand only basic knowledge of the content.	
Below Basic Content (0.0): Even with help, student has no understanding of the concept. Student demonstrates need for additional support in gaining the basic or prerequisite skill.	

Prerequisite skills for surface area and volume:

- Calculate central and interior angle measures for regular polygons
- Calculate the area and perimeter of polygons and circles
- Pythagorean theorem including special right triangles
- Solve equations.
- Right triangle trigonometry
- Operations and simplification of radicals

## 6. Suggested scope and sequence of Curriculum

### Lines and Angles:

- Understand and use definitions of points, lines, and planes
- Measure and construct segments and angles
- Identify and understand relationships formed by parallel lines and transversals
- Use the coordinate plane- points, lines, slopes of lines, transformations, midpoints, distances, parallel and perpendicular lines, various figures
- Construct, identify and use perpendicular and angle bisectors

### Geometric Reasoning and Logic:

- Write conditional statements, biconditional statements, and definitions
- Use deductive reasoning to make logical arguments
- Write proofs including two-column, narrative/paragraph, and flow chart

### Congruence

- Identify congruent objects using transformations, write congruence statements
- Classify triangles (right, obtuse, acute, scalene, isosceles, scalene, equilateral)
- Prove and apply the Isosceles Triangle Theorem
- Use triangle/angle relationships (Triangle Sum Theorem, Exterior Angle Theorem) to calculate angle measures
- Prove triangles congruent using Triangle Congruence Theorems including SSS, ASA, SAS, AAS, HL
- Understand why SSA and AAA do not prove triangle congruency
- Justify congruence of triangle parts using CPCTC (Corresponding Parts of Congruent Triangles are Congruent)

### Similarity, Right Triangle, and Trigonometry

- Understand basic concepts of similarity including defining similarity, identifying corresponding sides and angles, and setting up proportions using side lengths
- Identify opposite, adjacent, and hypotenuse sides of a right triangle
- Define trigonometric ratios – sine, cosine, and tangent and be able to represent each conceptually
- Use similarity postulates (AA, SSS, SAS) to identify similar triangles, and determine unknown side lengths and angle measures
- Understand and use the Pythagorean Theorem
- Use trigonometric ratios to determine side lengths and angle measures
- Identify special right triangles (30-60-90, and 45-45-90), and use to determine side lengths and angle measures
- Use scale factors to dilate polygons in the coordinate plane

### Surface Area and Volume

- Calculate the area and perimeter of polygons and circles
- Identify, describe, calculate angle measures and area of regular polygons
- Use a given formula to calculate the surface area of a figure with given dimensions
- Use formulas to calculate surface area and volume of solids
- Calculate dimensions of figures and objects using right triangles, trigonometry, and circle radii, circumference, and area

- Use area in geometric probability calculations.

### **Circles**

- Use the equation of a circle in the coordinate plane, derive the equation of a circle of given center and radius using the Pythagorean Theorem
- Derive and use the formulas for arc length and sector area
- Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

### **7. Suggested textbook(s), materials, equipment and resources**

No new common core books have been approved at this time. The Holt McDougal Mathematics *Explorations in Core Math for Geometry* has been used as a reference and sample text.

8. **Requirements satisfied** (“C” of the UC A-G requirements, one year of the three years of math required to graduate)

9. **Appendix** follows with a sample Geometry Understanding by Design Unit of Study for Program Goal 2 (Deductive Reasoning):

## Stage 1 Desired Results

<p><b>Established Goals</b></p> <p><b>Tam UHSD Program Goal #2:</b></p> <p>Students will use deductive reasoning to construct viable arguments and prove theorem about lines and angles</p> <p>CCSS Standard: <b>Mathematical Practice #3</b> Construct viable arguments and critique the reasoning of others</p> <p><b>HS-G.CO.9</b> Prove theorems about lines and angles. Theorems include vertical and congruent angles.</p>	<b>Transfer</b>	
	<p><i>Students will be able to independently use their learning to... use deductive reasoning prove a claim based on Triangle Congruency.</i></p> <p><i>Prove an argument using clear steps and having logical flow in a discussion with another student about math and in examples not pertaining to math</i></p> <p><i>Justify their reasoning in other mathematical problems (e.g solving algebraic problems)</i></p> <p>This unit introduces geometric reasoning and logic. Students will learn about various types of proofs including two column, flowchart and paragraph proofs. They will also begin to understand that proving an argument needs clear steps that others can logically follow. They will also learn how to assess if a "proof" is adequately proving its claim.</p>	
	<b>Meaning</b>	
	<p><b>UNDERSTANDINGS</b></p> <p><i>Students will understand that...</i></p> <p>Deductive logic is a powerful framework for making convincing arguments to establish conclusions.</p> <p>The truth value of different types of conditional statements (conditional and contrapositive are always the same, inverse and converse are always the same)</p> <p>The reasons why we do not use inductive reasoning in geometry, but where other disciplines rely on it.</p> <p>Reasoning and justification is vital in order to prove something as true.</p>	<p><b>ESSENTIAL QUESTIONS</b></p> <p><i>Students will keep considering...</i></p> <p>What value is there in using deductive reasoning?</p> <p>In what other disciplines does critical deductive thinking apply?</p> <p>How can you use examples to support or disprove a conjecture?</p> <p>How can we prove a statement as true?</p>

<b>Acquisition</b>	
<p><i>Students will know...</i></p> <p>The difference between inductive and deductive reasoning</p> <p>Why deductive reasoning is used to prove geometric theorems and know to not use inductive reasoning while proving theorems.</p> <p>The law of syllogism</p> <p><u>Lesson Specific Vocab</u></p> <p>Inductive Reasoning  Deductive Reasoning  Counterexample  Conditional  Converse  Inverse  Contrapositive  Syllogism  Biconditional  Properties of Congruency and Equality  Proof</p>	<p><i>Students will be skilled at</i></p> <p>Using inductive reasoning to identify patterns and make conjectures.</p> <p>Identifying the hypothesis and conclusion of conditional statements</p> <p>Given a conditional statement, writing the corresponding, converse, inverse, and contrapositive statements.</p> <p>Using the properties of equality and congruence to justify algebraic proofs</p> <p>How to create a logic chain</p> <p>Determining if a definition is good</p> <p>Disproving claims by finding a counter example</p>

Stage 2 - Evidence		
Code	Evaluative Criteria	Assessment Evidence
	Rubric	<p><b>PERFORMANCE TASK(S):</b>  <i>Students will show that they really understand by evidence of...</i>                      Creating conditional statements in if-then form, recognizing the hypothesis and conclusion, determine if a conditional is true or false, and provide a counter example to false statements.</p>
	Looking for Common Errors and Students who show little to no understanding	<p><b>OTHER EVIDENCE:</b>  <i>Students will show they have achieved Stage 1 goals by...</i>  <b>Exit Tickets</b>                      At the end of each day, students will fill out a short survey of that day's objective to see their level of understanding. Most tickets will also ask a meta-cognitive question about their learning process thus far.</p>
	ID Students who need more assistance	<p><b>Skills Quizzes</b>                      These formative assessments are slightly more formal about students level of understanding. These will assess the specific objectives of the lessons. Students will get their quizzes back.</p>
	Out of Points	<p><b>Test</b>                      A Unit Test has been made to address the objectives set out in this chapter.</p>

Stage 3 - Learning Plan		
<b>Code</b>	<p><i>Pre-assessment</i>                      The pre-assessment assesses the student's level of understanding on being able to determine the converse, inverse and contrapositive given a conditional statement as well as being able to use deductive reasoning to prove geometric theorems.</p>	
	<p><b>LEARNING EVENTS</b>  <i>Student success at transfer, meaning, and acquisition depends on...</i>  <b>CONSTRUCT VIABLE ARGUEMENTS</b>                      2.1 Inductive Reasoning                      -SWBAT (Student will be able to) use inductive reasoning to find and a pattern and make conjectures.                      -SWBAT disprove claims by finding a counter example.</p>	<p><i>Progress Monitoring</i>                      Exit Ticket: Given a number pattern, predict the next terms, then explain how they used inductive reasoning.</p>

	<p style="text-align: center;">2.2 Conditional Statements</p> <ul style="list-style-type: none"> <li>-SWBAT identify the hypothesis and conclusion of conditional (If, Then) statements</li> <li>-SWBAT determine the converse, inverse and contrapositive given a conditional statement.</li> <li>-SWBAT use truth values to determine the validity of a conditional statement</li> </ul> <p style="text-align: center;">2.3 Deductive Reasoning</p> <ul style="list-style-type: none"> <li>-SWBAT apply the Law of Syllogism (If <math>p \rightarrow q</math> and <math>q \rightarrow r</math> is true, then <math>p \rightarrow r</math> is true)</li> <li>-SWBAT create logic chains</li> </ul> <p style="text-align: center;">2.4 Biconditional Statements and Definitions</p> <ul style="list-style-type: none"> <li>-SWBAT write and analyze biconditional statements</li> <li>-SWBAT determine if a definition is good</li> </ul>	<p>Exit Ticket: Given a conditional statement they will ID the hypothesis and conclusion, the describe the validity of all the different types of statements.</p> <p>Exit Ticket: Given some statements in a random order student will connect a logic chain</p> <p>Skills Quiz on Inductive Reasoning and Conditional Statements</p> <p>Exit Ticket: Given a biconditional statement students will consider its validity and why it is biconditional and not just conditional.</p> <p>Skills Quiz on Deductive Reasoning, Biconditional Statements and possibly prior content, if many students needed to show improved understanding.</p> <p>Project: This is when the project would be introduced.</p> <p>Exit Ticket: Given an algebraic equations students will solve it and give the reason why for each step.</p>
--	---	--

	<p>PROVE THEOREMS ABOUT LINES AND ANGLES</p> <p>2.5 Algebraic Proof          -SWBAT use the properties of equality and congruence to justify algebraic proofs.</p> <p>2.6 Geometric Proof (Two Column)          -SWBAT use deductive reasoning to prove geometric theorems (e.g. linear pair theorem, overlapping angles theorem, vertical angles theorem)</p> <p>Review          -SWBAT review the past material to prepare for the Test</p>	<p>Exit Ticket: Students will fill in the blanks of a geometric two column proof and explain how they are using deductive reasoning,</p> <p>Unit Test: Students will take a test which is broken down by the objectives above.</p>
--	---	--