Energy, Momentum, and Relativity Review Sheet Solutions

1 answer: C

The force of gravity is proportional to the inverse of the square of distance between two masses. If the distance is changed from R to $4R$, the force is 1/16th as much. 400 N/16 = 25 N.

2 answer: C

$$F_g = \frac{Gm_1m_2}{r^2} \Rightarrow 3.4\times10^{-11} = \frac{(6.67\times10^{-11})(m^2)}{0.70^2}$$

$$m = \sqrt{\frac{0.70^2(3.4\times10^{-11})}{6.67\times10^{-11}}} = 0.50 \text{ kg}$$

3 answer: D

Newton's Law of Universal Gravitation shows that the force between two masses is inversely proportional to the square of the distance between the masses. The force on the comet is at a maximum at point A.

The kinetic energy of an orbiting comet is greatest when it is closest. This is where its potential energy is the least. Therefore, the comet's speed is maximum at point A.

4 answer: D

Kepler's three laws:

I. Planets orbit in elliptical paths.

II. Planets sweep out equal areas (sectors of the ellipse) in equal times.

III. The square of the orbital period is proportional to the cube of the orbital distance (or semi-major axis).

Only Kepler's 2nd law is expressed in statement D.

5 answer: B

$$\frac{T_B^{\ 2}}{T_A^{\ 2}} = \frac{R_B^{\ 3}}{R_A^{\ 3}} \Rightarrow T_B = \sqrt{\frac{T_A^{\ 2}\cdot R_B^{\ 3}}{R_A^{\ 3}}}$$

$$T_B = \sqrt[3]{\frac{(T)^2(4R)^3}{R^3}} = 8T$$

6 answer: A

$$KE_i = \tfrac{1}{2}mv^2 \quad KE_f = \tfrac{1}{2}(2m)(\tfrac{1}{2}v)^2 = \tfrac{1}{2}(\tfrac{1}{2}mv^2) = \tfrac{1}{2}KE_i$$

7 answer: C

$$F_g = mg = (2)(9.8) = 19.6 \text{ N}$$

$$W_{net} = F_1d_1\cos\theta_1 + F_2d_2\cos\theta_2 + F_3d_3\cos\theta_3$$

$$W = (19.6)(1.5)\cos 0° + (19.6)(15)\cos 90° + (19.6)(0.5)\cos 0°$$

$$W = 39.2 \text{ J}$$

8 answer: D

$$P = \frac{W}{t} = \frac{F_g d\cos\theta}{t} = \frac{(2 \text{ kg})(9.8 \text{ N/kg})(1.5 \text{ m})\cos 0°}{2.5 \text{ s}} = 11.8 \text{ W}$$

9 answer: A

$$Ft = m\Delta v = \Delta p$$

$$\Delta p = (2.5 \text{ kg})(9.8 \text{ N/kg})(4.0 \text{ s}) = 98 \text{ kg}\cdot\text{m/s}$$

10 answer: C

$$\Delta p = mv_f \Rightarrow 98 \text{ kg}\cdot\text{m/s} = (2.5 \text{ kg})(v_f) \Rightarrow v_f = 39.2 \text{ m/s}$$

$$KE_f = \tfrac{1}{2}mv_f^{\ 2} = \tfrac{1}{2}(2.5 \text{ kg})(39.2 \text{ m/s})^2 \Rightarrow KE_f = 1921 \text{ J}$$

11 answer: B

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$0 = (1.2 \text{ kg})(v_{1i}) + (1.8 \text{ kg})(2.0 \text{ m/s}) \Rightarrow v_{1i} = -3.0 \text{ m/s}$$

$$KE_f = \tfrac{1}{2}(1.2)(-3)^2 + \tfrac{1}{2}(1.8)(2)^2 = 9.0 \text{ J}$$

12 answer: D

$$Ft = m\Delta v \Rightarrow F(0.14 \text{ s}) = (1250 \text{ kg})(0 - 24 \text{ m/s})$$

$$|F| = |-2.14\times10^5 \text{ N}| = 2.14\times10^5 \text{ N}$$

13 answer: C

The egg's mass and its velocity at impact are unchanged by the foam pad, and the impulse on the egg is unchanged by the foam pad. However, the way in which the impulse is imparted does change – namely force is less while time is more, but the product is the same.

14 answer: C

$$I = \Delta p = m\Delta v = m(v_f - v_i)$$

$$I = (2 \text{ kg})(3 \text{ m/s} - (-5 \text{ m/s})) = 16 \text{ kg}\cdot\text{m/s}$$

15 answer: B

Newton's 3rd law says the force on each skater is equal. Since time of interaction must be equal, each skater gets equal and opposite impulse and change of momentum.

16 answer: D

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$$

$$(1)(6) + (5)(4) = (1+5)v_f \Rightarrow v_f = 4.33 \text{ m/s}$$

$$KE_i = \tfrac{1}{2}(1)(6^2) + \tfrac{1}{2}(5)(4^2) = 58 \text{ J}$$

$$KE_f = \tfrac{1}{2}(1+5)(4.33^2) = 56.3 \text{ J}$$

$$\% \text{ dissipated} = \frac{58 - 56.3}{58}\times 100\% = 2.9\%$$

17 answer: D

An elastic collision is defined as one in which no thermal energy is created, and all kinetic energy of the system is conserved. Of course, kinetic energy can be moved from one mass to another and still be conserved overall.

18 answer: A

$F_{sp} = k\Delta x$ balances mg

$$40 \text{ N} = k(0.6 \text{ m} - 0.4 \text{ m}) \Rightarrow k = 200 \text{ N/m}$$

$$EPE = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(200 \text{ N/m})(0.2 \text{ m})^2 = 4.0 \text{ J}$$

19 answer: A

$$KE_i = GPE_f = mgh$$

$$KE_i = (1.4 + 0.014)(9.8)(0.014) = 0.194 \text{ J}$$

20 answer: D

$$KE_i = \tfrac{1}{2}mv_i^{\ 2} \quad 0.194 = \tfrac{1}{2}(0.014 + 1.4)v_i^{\ 2}$$

$v_i = 0.524$ m/s this is v_f after the collision

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$$

$$(0.014)(v_{1i}) = (0.014 + 1.4)(0.524) \quad v_{1i} = 52.9 \text{ m/s}$$

21 answer: A

$\gamma = 1/\sqrt{1-v^2/c^2}$ solve this for v:

$$v = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{8^2}} = 0.992c$$

22 answer: D

There are two postulates to Special Theory: 1) all uniform frames agreed on the same laws of physics and 2) speed of light is the same for all frames. Only answer D is a postulate.

23 answer: A

The key difference between postulate and consequence is that the Special Theory is founded on two postulates (see above) but there are many consequences (tests, results). One result is that moving clocks appear to run slower than when at rest.

24 answer: D

$$\gamma = 1/\sqrt{1-v^2/c^2} = 1/\sqrt{1-(0.8c)^2/c^2} = 1.67$$

$$t = \gamma t_0 = 1.67(15) = 25 \text{ years} \Rightarrow \text{year: } 2019+25 = 2044$$

25 answer: B

$$\gamma = 1/\sqrt{1-v^2/c^2} = 1/\sqrt{1-(0.95c)^2/c^2} = 3.203$$

$$L = L_0/\gamma = 1.0/3.203 = 0.31 \text{ m}$$

26 answer: C

$$\gamma = 1/\sqrt{1-v^2/c^2} = 1/\sqrt{1-(0.6c)^2/c^2} = 1.25$$

$$E = \gamma mc^2 = 1.25mc^2 \quad (\text{and } E_0 = mc^2)$$

$$K = E - E_0 = (\gamma-1)mc^2 = (1.25-1)mc^2 = 0.25mc^2$$

27 answer: D

set centripetal force = gravitational force

$$\frac{mv^2}{R} = \frac{GmM}{R^2} \Rightarrow v = \sqrt{\frac{GM}{R}}$$

28 answer: A

$$\frac{T_B{}^2}{T_A{}^2} = \frac{R_B{}^3}{R_A{}^3} \Rightarrow R_B = \sqrt[3]{\frac{R_A{}^3 \cdot T_B{}^2}{T_A{}^2}} \Rightarrow R_B = \sqrt[3]{\frac{4.5^3 \cdot 9.6^2}{11.4^2}} = 4.0$$

so the orbital distance is 4.0×10^6m

29 answer: C

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$2(10) + 3(-5) = 2v_{1f} + 3v_{2f}$$

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

$$10 - (-5) = v_{2f} - v_{1f} \Rightarrow 15 + v_{1f} = v_{2f}$$

$$2(10) + 3(-5) = 2v_{1f} + 3(15 + v_{1f})$$

$$20 - 15 - 45 = 2v_{1f} + 3v_{1f}$$

$$v_{1f} = -8 \text{ m/s}; \; v_{2f} = 15 + -8 = 7 \text{ m/s}$$

30 answer: D

$$p = \gamma mv = 2mc$$

$$v = 2c/\gamma = 2c\sqrt{1-v^2/c^2}$$

$$v^2 = 4c^2(1-v^2/c^2) \Rightarrow 5v^2 = 4c^2 \Rightarrow v = \sqrt{\tfrac{4}{5}c^2} = 0.89c$$

31 answer: B

$$L = \frac{L_0}{\gamma} \Rightarrow L = L_0\sqrt{1-\left(\frac{v^2}{c^2}\right)} = (12.6 \text{ ly})\sqrt{1-\left(\frac{v^2}{c^2}\right)}$$

$$v = \frac{L}{t_0} = \frac{(12.6 \text{ ly})\sqrt{1-\left(\frac{v^2}{c^2}\right)}}{9.4 \text{ yr}} = (1.34c)\sqrt{1-\left(\frac{v^2}{c^2}\right)}$$

$$v = \sqrt{1.34^2(c^2-v^2)} \Rightarrow v = \left(\frac{1.34^2}{1+1.34^2}\right)c = 0.80c$$

32 (a)

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$F_g = \frac{(6.67\times10^{-11})(1.99\times10^{30})(0.107\times5.97\times10^{24})}{(1.52\times1.496\times10^{11})^2}$$

$$F_g = \boxed{1.64\times10^{21} \text{ N}}$$

(b)

$$T^2 = \left(\frac{4\pi^2}{GM}\right)R^3 = \left(\frac{4\pi^2}{G(1.99\times10^{30})}\right)(2.27\times10^{11})^3$$

$$T = 5.914\times10^7 \text{ s}$$

(c)

$$Sol = 24\text{h}\times\frac{3600\text{s}}{\text{h}} + 39 \text{ min}\times\frac{60\text{s}}{\text{min}} + 35 \text{ s} = 88{,}775 \text{ s}$$

$$T = 5.914\times10^7 \text{ s}\times\frac{1 \text{ Sol}}{88{,}775 \text{ s}} = \boxed{666 \text{ Sols}}$$

(d)

$$v = \frac{2\pi R}{T} = \frac{2\pi(2.28\times10^{11})}{5.914\times10^7} = 2.42\times10^4 \text{ m/s}$$

33 (a)

$$GPE = mgh = (90 \text{ N})(6.0 \text{ m}) = \boxed{540 \text{ J}}$$

(b) $W = GPE + TE \Rightarrow Fd\cos\theta = mgh + TE$

$$(70 \text{ N})(10 \text{ m}) = (90 \text{ N})(6.0 \text{ m}) + TE$$

$$TE = \boxed{160 \text{ J}}$$

(c) $TE = F_kd = \mu_k mg\cos\theta d$

$$160 \text{ J} = \mu_k(90 \text{ N})\cos 36.9^\circ(10 \text{ m})$$

$$\mu_k = \boxed{0.22}$$

34 $GPE_i = KE_f + TE \Rightarrow m_2gh = \frac{1}{2}(m_1+m_2)v^2 + \mu_k m_1gh$

$$2.0(9.8)(1.5) = \tfrac{1}{2}(3.0+2.0)v^2 + 0.15(3.0)(9.8)(1.5)$$

$$v = \boxed{3.02 \text{ m/s}}$$

35 (a)

$GPE_i = GPE_f + KE_f \Rightarrow mgh_1 = mgh_2 + \frac{1}{2}mv_2^{\,2}$

$(9.8)(17.5) = (9.8)(11) + \frac{1}{2}v_2^{\,2}$

$v_2 = \boxed{11.3\text{ m/s}}$

(b) $GPE_i = GPE_f + TE_f \Rightarrow mgh_1 = mgh_3 + TE$

$(50)(9.8)(17.5) = (50)(9.8)(4.5) + TE$

$TE = 50(9.8)(13) = \boxed{6{,}370\text{ J}}$

(c) $TE = F_k d \Rightarrow F_k = TE/d = 6370/10 \Rightarrow F_k = \boxed{637\text{ N}}$

36 (a)

$KE_i = EPE_f \Rightarrow \frac{1}{2}(m+M)v^2 = \frac{1}{2}kx^2$

$\frac{1}{2}(0.021+1.28)v^2 = \frac{1}{2}(230)(0.17)^2$

$v = \boxed{2.26\text{ m/s}}$

(b)

$mv_{1i} + M(0) = (m+M)v_f$

$0.021v_{1i} = (0.021+1.28)(2.26) \Rightarrow v_{1i} = \boxed{140\text{ m/s}}$

(c)

$\frac{1}{2}(m+M)v^2 = \frac{1}{2}kx^2 + \mu_k(m+M)gx$

$\frac{1}{2}(0.021+1.28)(2.26)^2 =$

$\frac{1}{2}(230)x^2 + 0.14(0.021+1.28)(9.8)x$

$115x^2 + 1.785x - 3.322 = 0$

$x = 0.162$ or $\boxed{16.2\text{ cm}}$

37 (a)

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

$1.4(+3.6) + 1.2(-1.6) = 1.4v_{1f} + 1.2(+4.0) \Rightarrow v_{1f} = \boxed{-1.2\text{ m/s}}$

(b)

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

$1.4(+3.6) + 1.2(-1.6) = 1.4v_{1f} + 1.2v_{2f}$

$5.04 - 1.92 = 3.12 = 1.4v_{1f} + 1.2v_{2f}$

$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$

$3.6 - (-1.6) = v_{2f} - v_{1f} \Rightarrow v_{2f} = v_{1f} + 5.2$

$3.12 = 1.4v_{1f} + 1.2(v_{1f} + 5.2)$

$2.6v_{1f} = 3.12 - 6.24$

$v_{1f} = \boxed{-1.2\text{ m/s}}$

$v_{2f} = -1.2 + 5.2 = \boxed{+4.0\text{ m/s}}$

(c)

$m_1v_{1i} + m_2v_{2i} = (m_1+m_2)v_f$

$(+1.4)(3.6) + (1.2)(-1.6) = (1.4+1.2)v_f \Rightarrow v_f = \boxed{+1.2\text{ m/s}}$

$KE_f - KE_i = \frac{1}{2}(2.6)(1.2)^2 - \left[\frac{1}{2}(1.4)(3.6)^2 - \frac{1}{2}(1.2)(-1.6)^2\right]$

$1.87 - 10.61 = \boxed{-8.74\text{ J}}$

38 (a)

To an Earth observer, 4.24 light years is the proper length, L_0, so the time to travel this proper length is

$t = \frac{L_0}{v} = \frac{(4.24\text{ 1y})}{0.75c} = \boxed{5.65\text{ years}}$

(b) Using the known dilated time, t, of 5.65 years, find the proper time, t_0, on the spacecraft

$t = \gamma t_0 = \frac{t_0}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} \Rightarrow t_0 = 5.65\sqrt{1-(0.75c)^2/c^2}$

$t_0 = \boxed{3.74\text{ years}}$

(c) To the spacecraft observer, the distance from Earth to the star is the contracted length, L

$L = \frac{L_0}{\gamma} \Rightarrow L = L_0\sqrt{1-\left(\frac{v^2}{c^2}\right)} = (4.24\text{ 1y})\sqrt{1-(0.75)^2}$

$L = \boxed{2.80\text{ 1y}}$

39

$GPE_i + KE_i = EPE_f$

$mg(d+x)\sin\theta + \frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$(1.85)(9.8)(0.72+x)\sin 25° + \frac{1}{2}(1.85)(0.75)^2 = \frac{1}{2}(160)x^2$

$80x^2 - 7.66x - 6.04 = 0 \Rightarrow x = \boxed{0.327\text{ m}}$