Blockday 5/19 2017

Honors Precalculus Preview of Rectilinear Motion. Name________________

A particle that can move in either direction along a coordinate line is said to be in rectilinear motion. The line might be n x-axis, y-axis, or a coordinate line inclined at some angle. For this worksheet, we will designate the coordinate line as the s-axis. We will assume that units are chosen for measuring distance and time and that we begin observing the motion of the particle at time t = 0. As the particle moves along the s-axis, its coordinate, s, will be some function of time, s = s(t). We will call s(t) the position function of the particle, and we call the graph of s versus t, the position versus time curve.

If the coordinate of a particle at time t_1 is $s(t_1)$ and the coordinate at a later time t_2 is $s(t_2)$, then $s(t_2)-s(t_1)$ is called the displacement of the particle over the time interval $[t_1,t_2]$. The displacement describes the change in position of the particle. We can tell from the sign of s when the particle is on the negative or positive side of the origin as it moves along the coordinate line.

In calculus, you will learn that the instantaneous velocity of a particle in rectilinear motion is the derivative of the position function. The velocity function will be $v(t)$, where v is the instantaneous velocity of the particle at time t. The sign of the velocity tells which way the particle is moving. The rate at which the instantaneous velocity of a particle changes with time is called the instantaneous acceleration. This happens to be the derivative of the velocity function. The acceleration function will be $a(t)$, where a is the instantaneous acceleration of the particle at time t.

A particle in rectilinear motion is speeding up when its velocity and acceleration have the same sign and slowing down when they have opposite signs. A particle is speeding up when its "speed" is increasing and is slowing down then its "speed" is decreasing. Speed is the absolute value of velocity.

In each problem, the position, velocity, and acceleration functions will be provided. Complete the following for each scenario presented.

a. Determine the position of the particle at t = 0.
b. Determine when the particle is stopped, traveling to the right, and traveling to the left.
c. Determine the position of the particle at each time it has stopped.
d. Sketch a schematic diagram to show the motion of the particle.
e. Determine the displacement of the particle over the time interval provided.
f. Determine the total distance traveled by the particle over the time interval provided.
g. Determine the intervals in which the particle is speeding up or slowing down.

Example: $s(t)=2t^3-24t^2+90t$ $v(t)=6t^2-48t+90$ $a(t)=12t-48$ on time interval $[0,9]$ units of measure will be feet and time.

Complete parts a-g for #1-4. Assume distances are measured in feet and time in seconds.

1.	2.	3.	4.
$s(t)=-t^3+18t^2-81t$	$s(t)=t^3-6t^2+9t$	$s(t)=4t^3-24t^2+21t-6$	$s(t)=-4t^2+12t+3$
$v(t)=-3t^2+36t-81$	$v(t)=3t^2-12t+9$	$v(t)=12t^2-48t+21$	$v(t)=-8t+12$
$a(t)=-6t+36$	$a(t)=6t-12$	$a(t)=24t-48$	$a(t)=-8$
For time $[0,10]$	For time $[0,5]$	For time $[0,9]$	For time $[0,3]$

1. $s(t) = -t^3 + 18t^2 - 81t$

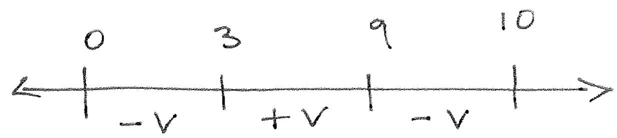
$v(t) = -3t^2 + 36t - 81$

$a(t) = -6t + 36$

$[0, 10]$

a. $s(0) = \boxed{0 \text{ feet}}$

b.



$v(t) = 0$

$-3t^2 + 36t - 81 = 0$

$-3(t^2 - 12t + 27) = 0$

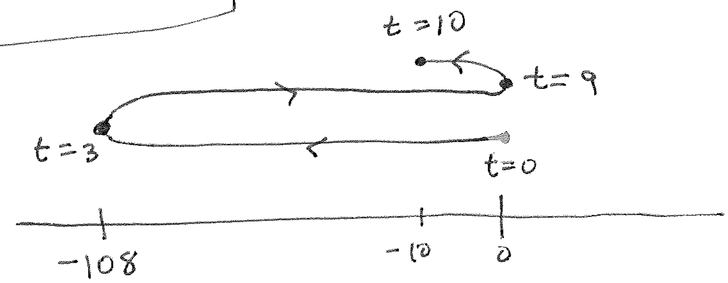
$-3(t-9)(t-3) = 0$

$t = 9 \quad t = 3$

Stopped at $t = 3$ seconds and 9 seconds
Travels left $0 < t < 3$ and $9 < t < 10$
Travels right $3 < t < 9$

c. $s(3) = -108$ ft
$s(9) = 0$ ft.

(d)



e. $s(10) - s(0)$

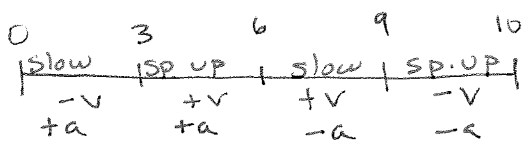
$-10 - 0$

displacement $\boxed{-10 \text{ ft.}}$

f. Total distance

$108 + 108 + 10 \quad \boxed{226 \text{ ft}}$

g. $-6(t-6)$



Speeds up: $3 < t < 6$ and $9 < t < 10$

slow down: $0 < t < 3$ and $6 < t < 9$

② $s(t) = t^3 - 6t^2 + 9t$ $\qquad [0, 5]$

$v(t) = 3t^2 - 12t + 9$

$a(t) = 6t - 12$

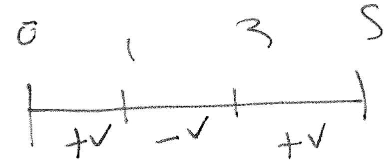
a. $s(0) = \boxed{0 \text{ ft}}$

b. $v(t) = 0$ when $0 = 3t^2 - 12t + 9$

$3(t^2 - 4t + 3)$

$3(t-3)(t-1)$

$t = 3$ or 1



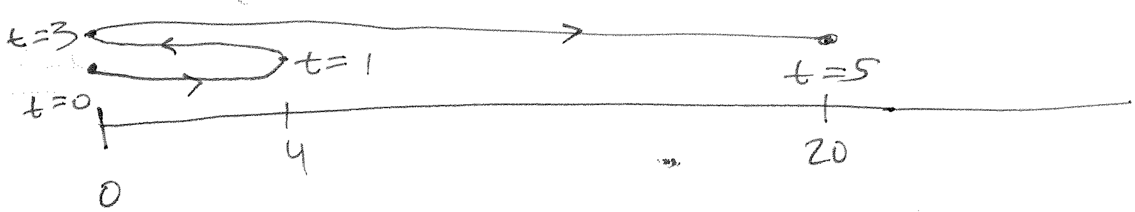
Stopped 1 second
3 seconds

travels right $0 < t < 1$
$3 < t < 5$

travels left $1 < t < 3$

c. $s(1) = 4$ ft
$s(3) = 0$ ft

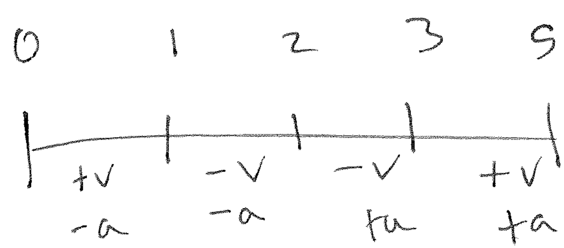
d.



e. $s(5) - s(0)$

$20 - 0$

$\boxed{20 \text{ ft}}$



f. $4 + 4 + 20$

$\boxed{28 \text{ ft}}$

g. slow down $0 < t < 1$ and $2 < t < 3$
speed up $1 < t < 2$ and $3 < t < 5$

$6(t-2)$

(3) $s(t) = 4t^3 - 24t^2 + 21t - 6$ $\quad [0, 9]$

$v(t) = 12t^2 - 48t + 21$

$a(t) = 24t - 48$

a. $s(0) = -6$ $\boxed{-6 \text{ ft.}}$

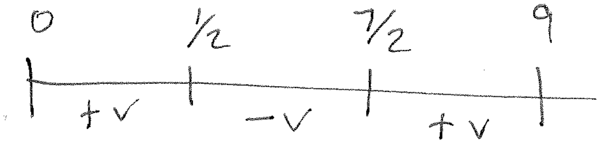
b. $v(t) = 0$

$0 = 12t^2 - 48t + 21$

$0 = 3(4t^2 - 16t + 7)$

$0 = 3(2t - 7)(2t - 1)$

$t = 1/2,\ 7/2$



$a(t) = 0$

$24t - 48 = 0$

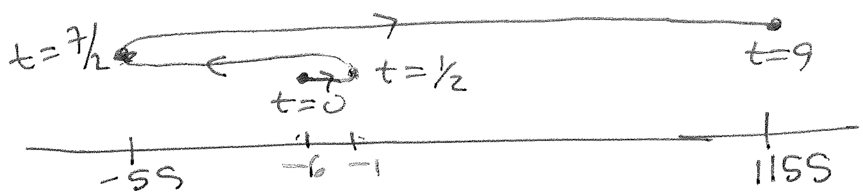
$t = 48/24 = 2$

Stopped at 1/2 second and 7/2 seconds
travels right $0 < t < 1/2$ and $7/2 < t < 9$
travels left $1/2 < t < 7/2$

c. $s(1/2) = -1$ ft

$s(7/2) = -55$ ft

d.



e. $s(9) - s(0) = \boxed{1161 \text{ ft.}}$

$1155 - -6$

f. $5 + 54 + 1210 = \boxed{1269 \text{ ft.}}$

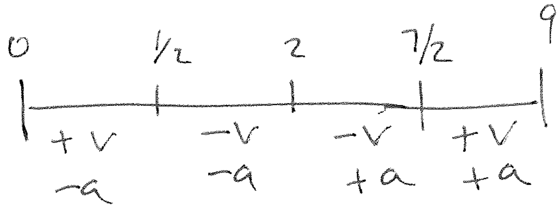
g. speeding up $1/2 < t < 2$

$7/2 < t < 9$

slowing down

$0 < t < 1/2$

$2 < t < 7/2$



(4) $s(t) = -4t^2 + 12t + 3$ $\quad [0, 3]$

$v(t) = -8t + 12$

$a(t) = -8$

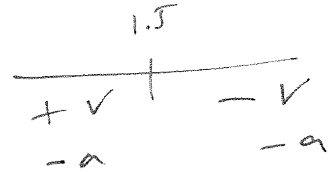
a. $S(0) = \boxed{3 \text{ ft.}}$

b. $v(t) = 0$

$-8t + 12 = 0$

$-8t = -12$

$t = 12/8 = 1.5$ seconds



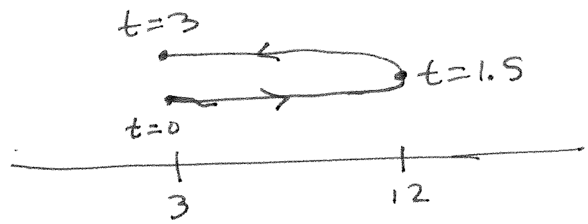
Stopped at 1.5 seconds

travels right $\quad 0 < t < 1.5$

travels left $\quad 1.5 < t < 3$

c. $S(1.5) = \boxed{12 \text{ ft.}}$

d. $\longrightarrow$



e. $s(3) - s(0)$

$3 - 3 = \boxed{0 \text{ ft}}$

f. $9 + 9 \quad \boxed{18 \text{ ft}}$

g. slow down $\quad 0 < t < 1.5$

speed up $\quad 1.5 < t < 3$