INTRODUCTION TO GRAPHING RATIONAL FUNCTIONS

Key Features
- Domain
- Holes, (if any)
- Vertical Asymptotes (if any)
- Horizontal Asymptotes (if any)
- Y-intercept (if any)
- X-intercepts (if any)

EXAMPLE: Analyze the graph of the following rational function

$$R(x)=\frac{3(x+5)(x-1)}{2(x+2)(x-1)}$$

Domain: All real numbers except -2 and 1 $\{x \mid x \in \mathbb{R}, x \neq -2, x \neq 1\}$

Holes: There is a hole at (1, 3) , the y-coordinate is found by canceling the factor that causes the hole, and substituting x = 1 into the resulting rational function.

$$R(x)=\frac{3(x+5)}{2(x+2)} \text{ substitute x = 1, then} \frac{3(1+5)}{2(1+2)}=\frac{3(6)}{2(3)}=3$$

Vertical Asymptotes: The zero of the factor in the denominator that cannot be "canceled" out. Thus x = -2 is the vertical asymptote because the factor $(x+2)$ cannot be "canceled" out.

Horizontal Asymptote: See special box below. In this case the degree on top is the same as degree on the bottom. Thus the y = 3/2 is the horizontal asymptote.

y-Intercept: Find R(0), $R(0)=\frac{3(0+5)}{2(0+2)}=\frac{3(5)}{2(2)}=\frac{15}{4}$ Thus y-intercept is (0, 3.75)

x-intercept: Solve $0=\frac{3(x+5)}{2(x+2)}$ this happens when $3(x+5)=0$, which is when $x=-5$ Thus x-intercept is (-5, 0)

Asymptotes are graphed using dotted lines! Holes are open circles. Vertical asymptotes are never crossed, but horizontal asymptotes can be crossed locally. To determine shape, test points into function and use the key features.

HORIZONTAL ASYMPTOTE
Determine the degree of the polynomial in numerator and polynomial in denominator.
Top Heavy (Higher degree in numerator) $\Rightarrow$ No horizontal asymptote. Could be other type.
Bottom Heavy (Higher degree in denominator) $\Rightarrow y=0$ is horizontal asymptote
Same Degree Top & Bottom $\Rightarrow y=\frac{\textit{lead coefficient}}{\textit{lead coefficient}}$ is horizontal asymptote