

## Binomial Expansions

In Section 4.2, you used Pascal's Triangle to find binomial expansions. The table shows that the coefficients in the expansion of  $(a + b)^n$  correspond to combinations.

	$n$	Pascal's Triangle as Numbers	Pascal's Triangle as Combinations	Binomial Expansion
0th row	0	1	${}_0C_0$	$(a + b)^0 = 1$
1st row	1	1 1	${}_1C_0$ ${}_1C_1$	$(a + b)^1 = 1a + 1b$
2nd row	2	1 2 1	${}_2C_0$ ${}_2C_1$ ${}_2C_2$	$(a + b)^2 = 1a^2 + 2ab + 1b^2$
3rd row	3	1 3 3 1	${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$	$(a + b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$

The results in the table are generalized in the **Binomial Theorem**.

### Core Concept

#### The Binomial Theorem

For any positive integer  $n$ , the binomial expansion of  $(a + b)^n$  is  
 $(a + b)^n = {}_n C_0 a^n b^0 + {}_n C_1 a^{n-1} b^1 + {}_n C_2 a^{n-2} b^2 + \dots + {}_n C_n a^0 b^n$ .

Notice that each term in the expansion of  $(a + b)^n$  has the form  ${}_n C_r a^{n-r} b^r$ , where  $r$  is an integer from 0 to  $n$ .

Two methods To Explore:

- Method 1 – Use the formula in theorem with precision
- Method 2 – Apply the structure of formula in theorem and problem solve

#### TEACHER EXAMPLE 1 (showing both methods):

Use the Binomial Theorem to write the expansion of  $(x^2 + y)^3$

**TEACHER EXAMPLE 2:** Use the Binomial Theorem to write the expansion of  $(x+2y)^4$   
(method 1)

**STUDENT YOU TRY IT: (Method 1)**

Use the Binomial Theorem to write the expansion of  $(x-3)^5$

**TEACHER EXAMPLE 3:** Use the binomial theorem to write the expansion of  $(5x-y)^3$   
(method 2)

SET-UP:

**STUDENT YOU TRY IT: (Method 2):** Use the binomial theorem to write the expansion of

$(2x-y)^4$  (method 2 – set-up formula's structure)

**TEACHER EXAMPLE 4 : (Method 1 – precision with formula)**

Find the coefficient of  $x^4$  in the expansion of  $(3x+2)^{10}$

**Student You Try It :** (Method 1 – precision with formula)

Find the coefficient of  $x^5$  in the expansion of  $(x-3)^7$

**TEACHER EXAMPLE 5 : (Method 2)**

Find the coefficient of  $x^3$  in the expansion of  $(2x-7)^4$

**Student You Try It :** (Method 2)

Find the coefficient of  $x^3$  in the expansion of  $(2x+5)^8$