

Peach

Question 4

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \leq t \leq 80$ seconds, as shown in the table above.

- (a) Find the average acceleration of rocket A over the time interval $0 \leq t \leq 80$ seconds. Indicate units of measure.
- (b) Using correct units, explain the meaning of $\int_{10}^{70} v(t) dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t) dt$.
- (c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

- (a) Average acceleration of rocket A is

$$\frac{v(80) - v(0)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft/sec}^2$$

- (b) Since the velocity is positive, $\int_{10}^{70} v(t) dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds.

A midpoint Riemann sum is

$$\begin{aligned} &20[v(20) + v(40) + v(60)] \\ &= 20[22 + 35 + 44] = 2020 \text{ ft} \end{aligned}$$

- (c) Let $v_B(t)$ be the velocity of rocket B at time t .

$$v_B(t) = \int \frac{3}{\sqrt{t+1}} dt = 6\sqrt{t+1} + C$$

$$2 = v_B(0) = 6 + C$$

$$v_B(t) = 6\sqrt{t+1} - 4$$

$$v_B(80) = 50 > 49 = v(80)$$

Rocket B is traveling faster at time $t = 80$ seconds.

Units of ft/sec^2 in (a) and ft in (b)

1 : answer

3 : $\left\{ \begin{array}{l} 1 : \text{explanation} \\ 1 : \text{uses } v(20), v(40), v(60) \\ 1 : \text{value} \end{array} \right.$

4 : $\left\{ \begin{array}{l} 1 : 6\sqrt{t+1} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{finds } v_B(80), \text{ compares to } v(80), \\ \text{and draws a conclusion} \end{array} \right.$

1 : units in (a) and (b)

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Question 5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t , where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \leq t \leq 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

- (a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t) dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t) dt$ in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t) dt$? Give a reason for your answer.

(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft
 Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.

(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$
 $\left.\frac{dV}{dt}\right|_{t=5} = 4\pi(30)^2(2) = 7200\pi$ ft³/min

(c) $\int_0^{12} r'(t) dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$
 $= 19.3$ ft
 $\int_0^{12} r'(t) dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.

(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t) dt$.

Units of ft³/min in part (b) and ft in part (c)

2 : $\left\{ \begin{array}{l} 1 : \text{estimate} \\ 1 : \text{conclusion with reason} \end{array} \right.$

3 : $\left\{ \begin{array}{l} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{array} \right.$

2 : $\left\{ \begin{array}{l} 1 : \text{approximation} \\ 1 : \text{explanation} \end{array} \right.$

1 : conclusion with reason

1 : units in (b) and (c)

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Question 3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ ($^{\circ}\text{C}$)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius ($^{\circ}\text{C}$), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

- (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find $\int_0^8 T'(x) dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x) dx$ in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

(a)
$$\frac{T(8) - T(6)}{8 - 6} = \frac{55 - 62}{2} = -\frac{7}{2}^{\circ}\text{C/cm}$$

1 : answer

(b)
$$\frac{1}{8} \int_0^8 T(x) dx$$

Trapezoidal approximation for $\int_0^8 T(x) dx$:

$$A = \frac{100 + 93}{2} \cdot 1 + \frac{93 + 70}{2} \cdot 4 + \frac{70 + 62}{2} \cdot 1 + \frac{62 + 55}{2} \cdot 2$$

Average temperature $\approx \frac{1}{8}A = 75.6875^{\circ}\text{C}$

3 : $\left\{ \begin{array}{l} 1 : \frac{1}{8} \int_0^8 T(x) dx \\ 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{array} \right.$

(c)
$$\int_0^8 T'(x) dx = T(8) - T(0) = 55 - 100 = -45^{\circ}\text{C}$$

The temperature drops 45°C from the heated end of the wire to the other end of the wire.

2 : $\left\{ \begin{array}{l} 1 : \text{value} \\ 1 : \text{meaning} \end{array} \right.$

(d) Average rate of change of temperature on $[1, 5]$ is $\frac{70 - 93}{5 - 1} = -5.75$.

Average rate of change of temperature on $[5, 6]$ is $\frac{62 - 70}{6 - 5} = -8$.

No. By the MVT, $T'(c_1) = -5.75$ for some c_1 in the interval $(1, 5)$ and $T'(c_2) = -8$ for some c_2 in the interval $(5, 6)$. It follows that T' must decrease somewhere in the interval (c_1, c_2) . Therefore T'' is not positive for every x in $[0, 8]$.

2 : $\left\{ \begin{array}{l} 1 : \text{two slopes of secant lines} \\ 1 : \text{answer with explanation} \end{array} \right.$

Units of $^{\circ}\text{C/cm}$ in (a), and $^{\circ}\text{C}$ in (b) and (c)

1 : units in (a), (b), and (c)

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Question 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	-20	-30	-20	-14	-10	0	10
$a(t)$ (ft/sec ²)	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \leq t \leq 60$ seconds, the car's velocity v , measured in feet per second, and acceleration a , measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)| dt$ in terms of the car's motion. Approximate

$\int_{30}^{60} |v(t)| dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_0^{30} a(t) dt$ in terms of the car's motion. Find the exact value

of $\int_0^{30} a(t) dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

(a) $\int_{30}^{60} |v(t)| dt$ is the distance in feet that the car travels from $t = 30$ sec to $t = 60$ sec.

Trapezoidal approximation for $\int_{30}^{60} |v(t)| dt$:

$$A = \frac{1}{2}(14 + 10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185 \text{ ft}$$

(b) $\int_0^{30} a(t) dt$ is the car's change in velocity in ft/sec from $t = 0$ sec to $t = 30$ sec.

$$\begin{aligned} \int_0^{30} a(t) dt &= \int_0^{30} v'(t) dt = v(30) - v(0) \\ &= -14 - (-20) = 6 \text{ ft/sec} \end{aligned}$$

(c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.

(d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.

Units of ft in (a) and ft/sec in (b)

2 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{value} \end{cases}$

2 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{value} \end{cases}$

2 : $\begin{cases} 1 : v(35) < -5 < v(50) \\ 1 : \text{Yes; refers to IVT or hypotheses} \end{cases}$

2 : $\begin{cases} 1 : v(0) = v(25) \\ 1 : \text{Yes; refers to MVT or hypotheses} \end{cases}$

1 : units in (a) and (b)

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2008 SCORING GUIDELINES (Form B)

Question 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin(\sqrt{t+10})$ for $0 \leq t \leq 120$ minutes.

- (a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.
- (b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.
- (c) The scientist proposes the function f , given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.
- (d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \leq t \leq 60$ minutes. Does this value indicate that the water must be diverted?

(a)
$$\frac{(0+7)}{2} \cdot 8 + \frac{(7+8)}{2} \cdot 6 + \frac{(8+2)}{2} \cdot 8 + \frac{(2+0)}{2} \cdot 2$$

$$= 115 \text{ ft}^2$$

(b)
$$\frac{1}{120} \int_0^{120} 115v(t) dt$$

$$= 1807.169 \text{ or } 1807.170 \text{ ft}^3/\text{min}$$

(c)
$$\int_0^{24} 8\sin\left(\frac{\pi x}{24}\right) dx = 122.230 \text{ or } 122.231 \text{ ft}^2$$

(d) Let C be the cross-sectional area approximation from part (c). The average volumetric flow is

$$\frac{1}{20} \int_{40}^{60} C \cdot v(t) dt = 2181.912 \text{ or } 2181.913 \text{ ft}^3/\text{min}.$$

Yes, water must be diverted since the average volumetric flow for this 20-minute period exceeds $2100 \text{ ft}^3/\text{min}$.

1 : trapezoidal approximation

3 : $\left\{ \begin{array}{l} 1 : \text{limits and average value} \\ \quad \text{constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

3 : $\left\{ \begin{array}{l} 1 : \text{volumetric flow integral} \\ 1 : \text{average volumetric flow} \\ 1 : \text{answer with reason} \end{array} \right.$