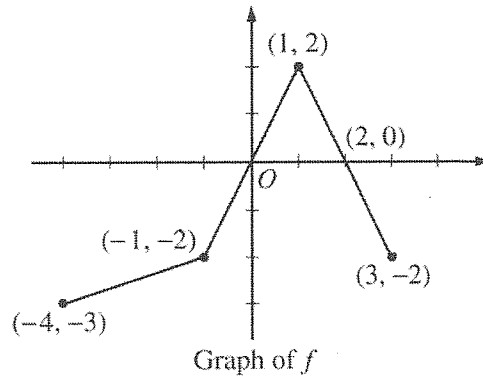


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2005 SCORING GUIDELINES (Form B)

Question 4

The graph of the function f above consists of three line segments.



(a) Let g be the function given by $g(x) = \int_{-4}^x f(t) dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x -coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_x^3 f(t) dt$. Find all values of x in the closed interval $-4 \leq x \leq 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

(a) $g(-1) = \int_{-4}^{-1} f(t) dt = -\frac{1}{2}(3)(5) = -\frac{15}{2}$
 $g'(-1) = f(-1) = -2$
 $g''(-1)$ does not exist because f is not differentiable at $x = -1$.

3 : $\begin{cases} 1 : g(-1) \\ 1 : g'(-1) \\ 1 : g''(-1) \end{cases}$

(b) $x = 1$
 $g' = f$ changes from increasing to decreasing at $x = 1$.

2 : $\begin{cases} 1 : x = 1 \text{ (only)} \\ 1 : \text{reason} \end{cases}$

(c) $x = -1, 1, 3$

2 : correct values
 $\langle -1 \rangle$ each missing or extra value

(d) h is decreasing on $[0, 2]$
 $h' = -f < 0$ when $f > 0$

2 : $\begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$

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2008 SCORING GUIDELINES (Form B)

Question 4

The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4+t^2} dt$ and $g(x) = f(\sin x)$.

- (a) Find $f'(x)$ and $g'(x)$.
- (b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.
- (c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \leq x \leq \pi$. Justify your answer.

(a) $f'(x) = 3\sqrt{4 + (3x)^2}$

$$g'(x) = f'(\sin x) \cdot \cos x$$

$$= 3\sqrt{4 + (3\sin x)^2} \cdot \cos x$$

4 : $\begin{cases} 2 : f'(x) \\ 2 : g'(x) \end{cases}$

(b) $g(\pi) = 0, g'(\pi) = -6$
 Tangent line: $y = -6(x - \pi)$

2 : $\begin{cases} 1 : g(\pi) \text{ or } g'(\pi) \\ 1 : \text{tangent line equation} \end{cases}$

(c) For $0 < x < \pi$, $g'(x) = 0$ only at $x = \frac{\pi}{2}$.

$$g(0) = g(\pi) = 0$$

$$g\left(\frac{\pi}{2}\right) = \int_0^3 \sqrt{4+t^2} dt > 0$$

The maximum value of g on $[0, \pi]$ is

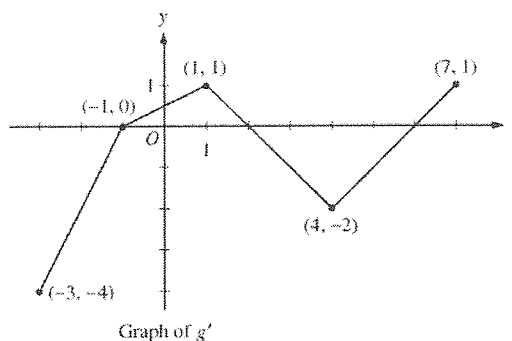
$$\int_0^3 \sqrt{4+t^2} dt.$$

3 : $\begin{cases} 1 : \text{sets } g'(x) = 0 \\ 1 : \text{justifies maximum at } \frac{\pi}{2} \\ 1 : \text{integral expression for } g\left(\frac{\pi}{2}\right) \end{cases}$

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2008 SCORING GUIDELINES (Form B)

Question 5

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g' , the derivative of g , is shown above for $-3 \leq x \leq 7$.



- (a) Find the x -coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.
- (b) Find the absolute maximum value of g on the interval $-3 \leq x \leq 7$. Justify your answer.
- (c) Find the average rate of change of $g(x)$ on the interval $-3 \leq x \leq 7$.
- (d) Find the average rate of change of $g'(x)$ on the interval $-3 \leq x \leq 7$. Does the Mean Value Theorem applied on the interval $-3 \leq x \leq 7$ guarantee a value of c , for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

- (a) g' changes from increasing to decreasing at $x = 1$;
 g' changes from decreasing to increasing at $x = 4$.

Points of inflection for the graph of $y = g(x)$ occur at $x = 1$ and $x = 4$.

- (b) The only sign change of g' from positive to negative in the interval is at $x = 2$.

$$g(-3) = 5 + \int_2^{-3} g'(x) dx = 5 + \left(-\frac{3}{2}\right) + 4 = \frac{15}{2}$$

$$g(2) = 5$$

$$g(7) = 5 + \int_2^7 g'(x) dx = 5 + (-4) + \frac{1}{2} = \frac{3}{2}$$

The maximum value of g for $-3 \leq x \leq 7$ is $\frac{15}{2}$.

(c)
$$\frac{g(7) - g(-3)}{7 - (-3)} = \frac{\frac{3}{2} - \frac{15}{2}}{10} = -\frac{3}{5}$$

(d)
$$\frac{g'(7) - g'(-3)}{7 - (-3)} = \frac{1 - (-4)}{10} = \frac{1}{2}$$

No, the MVT does *not* guarantee the existence of a value c with the stated properties because g' is not differentiable for at least one point in $-3 < x < 7$.

2 : $\begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

3 : $\begin{cases} 1 : \text{identifies } x = 2 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{maximum value and justification} \end{cases}$

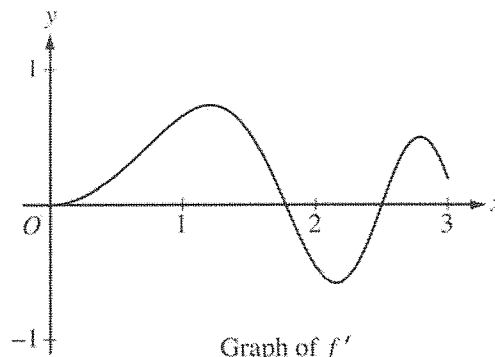
2 : $\begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{average value of } g'(x) \\ 1 : \text{answer "No" with reason} \end{cases}$

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2006 SCORING GUIDELINES (Form B)

Question 2

Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f' , the first derivative of f , given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown above.



- (a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.
- (b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.
- (c) Write an equation for the line tangent to the graph of f at $x = 2$.

(a) On the interval $1.7 < x < 1.9$, f' is decreasing and thus f is concave down on this interval.

2 : { 1 : answer
1 : reason

(b) $f'(x) = 0$ when $x = 0, \sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}, \dots$
On $[0, 3]$ f' changes from positive to negative only at $\sqrt{\pi}$. The absolute maximum must occur at $x = \sqrt{\pi}$ or at an endpoint.

3 : { 1 : identifies $\sqrt{\pi}$ and 3 as candidates
- or -
indicates that the graph of f increases, decreases, then increases
1 : justifies $f(\sqrt{\pi}) > f(3)$
1 : answer

$$f(0) = 5$$

$$f(\sqrt{\pi}) = f(0) + \int_0^{\sqrt{\pi}} f'(x) dx = 5.67911$$

$$f(3) = f(0) + \int_0^3 f'(x) dx = 5.57893$$

This shows that f has an absolute maximum at $x = \sqrt{\pi}$.

(c) $f(2) = f(0) + \int_0^2 f'(x) dx = 5.62342$

$$f'(2) = e^{-0.5} \sin(4) = -0.45902$$

$$y - 5.623 = (-0.459)(x - 2)$$

4 : { 2 : $f(2)$ expression
1 : integral
1 : including $f(0)$ term
1 : $f'(2)$
1 : equation

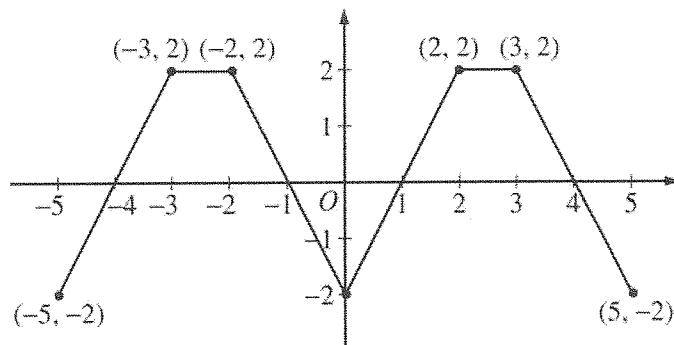
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Question 3

The graph of the function f shown above consists of six line segments. Let g be the function given by

$$g(x) = \int_0^x f(t) dt.$$

- (a) Find $g(4)$, $g'(4)$, and $g''(4)$.
 (b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.



Graph of f

- (c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f . Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

(a) $g(4) = \int_0^4 f(t) dt = 3$

$$g'(4) = f(4) = 0$$

$$g''(4) = f'(4) = -2$$

$$3 : \begin{cases} 1 : g(4) \\ 1 : g'(4) \\ 1 : g''(4) \end{cases}$$

- (b) g has a relative minimum at $x = 1$ because $g' = f$ changes from negative to positive at $x = 1$.

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$$

- (c) $g(0) = 0$ and the function values of g increase by 2 for every increase of 5 in x .

$$g(10) = 2g(5) = 4$$

$$\begin{aligned} g(108) &= \int_0^{105} f(t) dt + \int_{105}^{108} f(t) dt \\ &= 21g(5) + g(3) = 44 \end{aligned}$$

$$g'(108) = f(108) = f(3) = 2$$

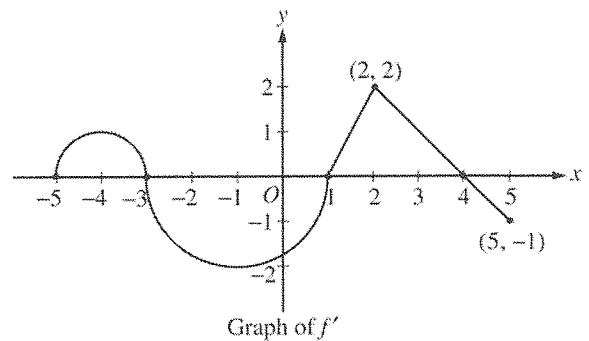
An equation for the line tangent to the graph of g at $x = 108$ is $y - 44 = 2(x - 108)$.

$$4 : \begin{cases} 1 : g(10) \\ 3 : \begin{cases} 1 : g(108) \\ 1 : g'(108) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$$

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2007 SCORING GUIDELINES (Form B)

Question 4

Let f be a function defined on the closed interval $-5 \leq x \leq 5$ with $f(1) = 3$. The graph of f' , the derivative of f , consists of two semicircles and two line segments, as shown above.



- For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
- For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
- Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
- Find the absolute minimum value of $f(x)$ over the closed interval $-5 \leq x \leq 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3, 1, 4$
 f' changes from positive to negative at -3 and 4 .
 Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

2 : { 1 : x-values
 1 : justification

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4, -1$, and 2 . Thus, the graph of f has points of inflection when $x = -4, -1$, and 2 .

2 : { 1 : x-values
 1 : justification

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

2 : { 1 : intervals
 1 : explanation

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

3 : { 1 : identifies $x = 1$ as a candidate
 1 : considers endpoints
 1 : value and explanation

$$f(-5) = 3 + \int_1^{-5} f'(x) dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^5 f'(x) dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.