

Consecutive Sums

Date: _____ Period: _____

A sequence of two or more *natural* numbers is **consecutive** if each number is one more than the previous number.

For example, the numbers **2, 3, and 4** are **consecutive**. The numbers 8, 9, 10, and 11 are consecutive. The numbers 23 and 24 are consecutive.

On the other hand, the numbers **6, 8, and 10** are **not consecutive**, because each number is *two* more than the previous number. Also, a single number by itself is not consecutive.

A **consecutive sum** is a sum of a sequence of consecutive numbers. So each expression below is a consecutive sum:

$$2 + 3 + 4$$

$$8 + 9 + 10 + 11$$

$$23 + 24$$

These examples illustrate how to express 9, 38, and 47 as consecutive sums. For this activity, you will only use **natural numbers**, which are positive integers that do not include 0.

Part I: Consecutive Sums for 1 through 35

Find a consecutive sum for each of the numbers from 1 through 35. So, the example above of $2 + 3 + 4 = 9$ may be used for 9. You may use as many consecutive numbers as you wish for each sum. Make a specific note of any numbers that might be impossible to write as a consecutive sum.

On your poster:

- Write your sums from 1 through 35.
- Write at least 3 conjectures or other patterns that you think will always be true for consecutive sums.
- Write a conjecture about when it is impossible to write a number as a consecutive sum. You might need to go beyond 35 to gather data for this.

Part II: Consecutive Sums using 3 Natural Numbers

Josh made a list of conjectures for consecutive sums that can be made using 3 numbers, like $2 + 3 + 4 = 9$. Your team needs to determine if his conjectures are correct. To do so, you will need to come up with several consecutive sums that only use 3 numbers. On a separate piece of paper, write an explanation about why you agree or disagree with each statement.

Josh's Conjectures:

- A consecutive sum of 3 natural numbers is always **positive**.
- A consecutive sum of 3 natural numbers is always a **multiple of 3**.
- A consecutive sum of 3 natural numbers is always an **odd**.

Part III: Consecutive Sums using 2 Natural Numbers

Jane made this conjecture: **Any odd number greater than 1 can be written as the sum of two consecutive natural numbers.**

Is Jane correct? If so, write out why she is correct. If she is not correct, write out a *counterexample* that proves that she is wrong. Write this on the same paper that you used for Part II.