BC Calculus
10-#11 Logistic Differential Equations

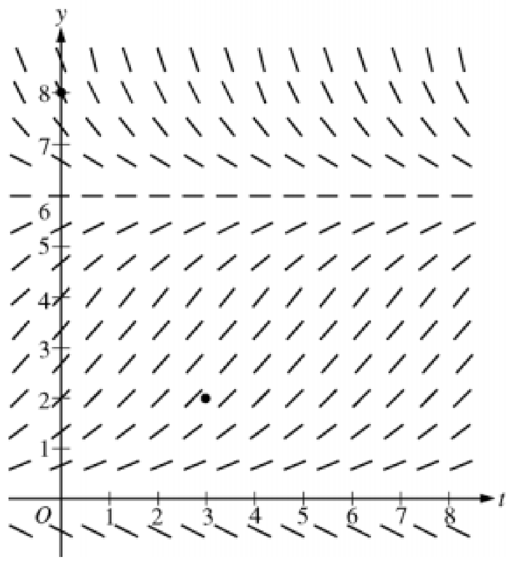
Name: ______________________________
Period:

1. Suppose the population of bears in a national park grows according to the logistic differential equation $\frac{dP}{dt} = 5P - 0.002P^2$, where P is the number of bears at time t in years for $t \geq 0$. $P(0) = 100$		
a) Find the carrying capacity.	b) Find $\lim_{t \to \infty} P(t)$	c) What is the range of the solutions curve?
d) For what values of P is the solutions curve increasing? Decreasing?	e) For what values of P is the solutions curve concave up? Concave down?	f) How many bears are in the park when the population of bears is growing fastest?
g) Sketch the solution curve.	h) If $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$ then $y = \frac{L}{1 + be^{-kt}}$. Use these formulas and the initial condition to find the solution curve. Compare the graph of your solution and the sketch you made in part (g).	

2. Suppose the population of bears in a national park grows according to the logistic differential equation $\frac{dP}{dt} = 5P - 0.002P^2$, where P is the number of bears at time t in years for $t \geq 0$. $P(0) = 4000$		
a) Find the carrying capacity.	b) Find $\lim_{t \to \infty} P(t)$	c) What is the range of the solutions curve?
d) For what values of P is the solutions curve increasing? Decreasing?	e) For what values of P is the solutions curve concave up? Concave down?	

f) Sketch the solution curve.	g) If $\frac{dy}{dt}=ky\left(1-\frac{y}{L}\right)$ then $y=\frac{L}{1+be^{-kt}}$. Use these formulas and the initial condition to find the solution curve. Compare the graph of your solution and the sketch you made in part (f).

3.	Suppose a rumor is spreading through a dance at a rate modeled by the logistic differential equation $\frac{dP}{dt}=P\left(3-\frac{P}{2000}\right)$. What is $\lim_{t\to\infty} P(t)$? What does this number represent in the context of this problem?
4.	The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt}=P\left(2-\frac{P}{5000}\right)$, where the initial population $P(0)=3,000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$? (A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000
5.	Suppose a population of wolves grows according to the logistic differential equation $\frac{dP}{dt}=3P-0.01P^2$, where P is the number of wolves at time t in years. Which of the following statements are true? I. $\lim_{t\to\infty} P(t)=300$ II. The growth rate of the wolf population is greatest at $P=150$. III. If $P>300$, the population of wolves is increasing. (A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III
6.	The rate of change, $\frac{dP}{dt}$, of the number of people on an ocean beach is modeled by a logistic differential equation. The maximum number of people allowed on the beach is 1200. At 10 A.M., the number of people on the beach is 200 and is increasing at the rate of 400 people per hour. Which of the following differential equations describes the situation? (A) $\frac{dP}{dt}=\frac{1}{400}(1200-P)+200$ (B) $\frac{dP}{dt}=\frac{2}{5}(1200-P)$ (C) $\frac{dP}{dt}=\frac{1}{500}P(1200-P)$ (D) $\frac{dP}{dt}=\frac{1}{400}P(1200-P)$ (E) $\frac{dP}{dt}=400P(1200-P)$
7.	Let k be a positive constant. Which of the following is a logistic differential equation? (A) $\frac{dy}{dt}=kt$ (B) $\frac{dy}{dt}=ky$ (C) $\frac{dy}{dt}=kt(1-t)$ (D) $\frac{dy}{dt}=ky(1-t)$ (E) $\frac{dy}{dt}=ky(1-y)$

8.	Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$. (a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$. (b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$. (c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$. (d) What is the range of f for $t \geq 0$?
	

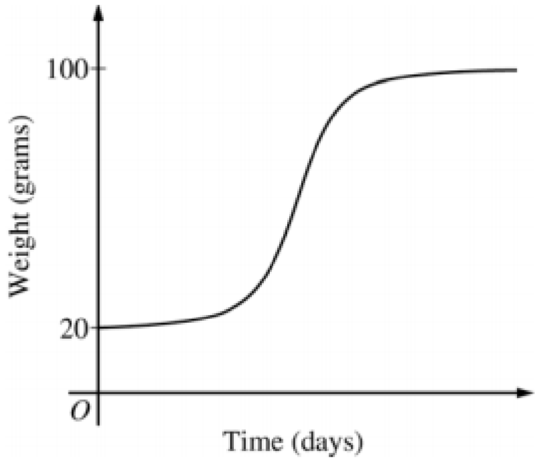
9. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B <u>cannot</u> resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.