BC Calculus | Name:______________________________
Logistic Growth | 10-11 Notes

In exponential growth, we assume that the rate of increase (or decrease) of a population at any time t is directly proportional to the population P. In other words, $\frac{dP}{dt} = kP$. However, in many situations population growth levels off and approaches a limiting number L (the carrying capacity) because of limited resources. In this situation, the rate of increase (or decrease) is directly proportional to both P and $L - P$. This type of growth is called **logistic growth**. It is modeled by the differential equation $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$.

If we find $\frac{d^2P}{dt^2}$, we can find out an important fact about the time when P is growing the fastest.

Ex 1) The population $P(t)$ of fish in a lake satisfies the logistic differential equation $\frac{dP}{dt} = 3P - \frac{P^2}{6000}$, where t is measured in years.

a) Rewrite $\frac{dP}{dt} = 3P - \frac{P^2}{6000}$ in the form $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$.

b) $\lim_{t \to \infty} P(t) =$

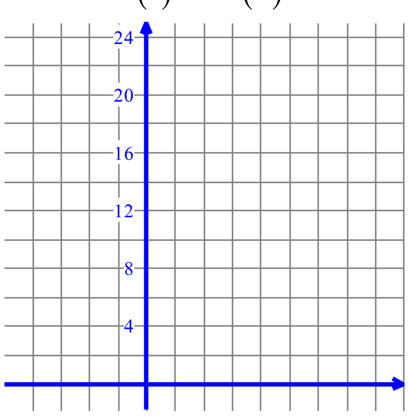
This value is also called the

____

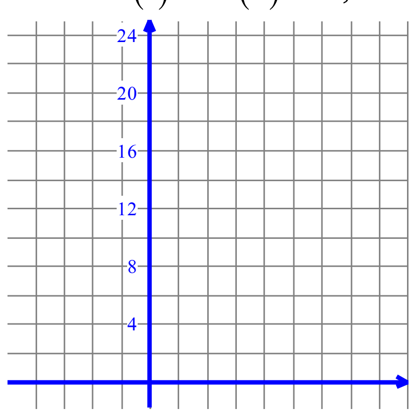
c) Solve $\frac{dP}{dt} = 0$ and make a sign chart. Interpret the results.

d) Find $\frac{d^2P}{dt^2}$, solve $\frac{d^2P}{dt^2} = 0$, make a sign chart and interpret results.

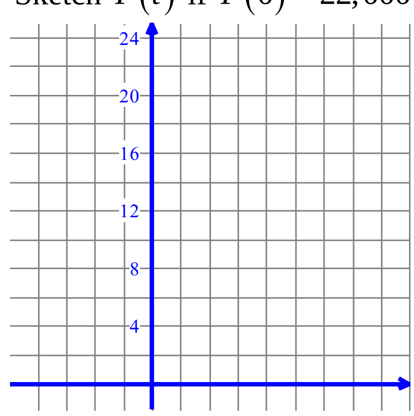
e) Sketch $P(t)$ if $P(0) = 4000$



f) Sketch $P(t)$ if $P(0) = 10,000$



g) Sketch $P(t)$ if $P(0) = 22,000$



Determine the intervals of P where the $P(t)$ is concave up or concave down. Assume $t \geq 0$.

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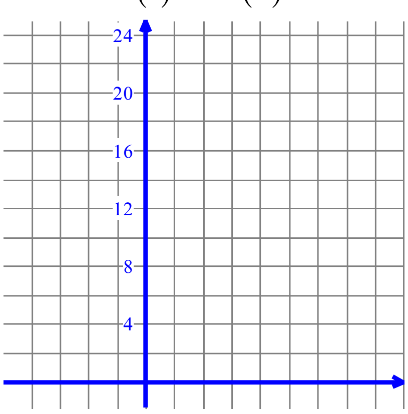

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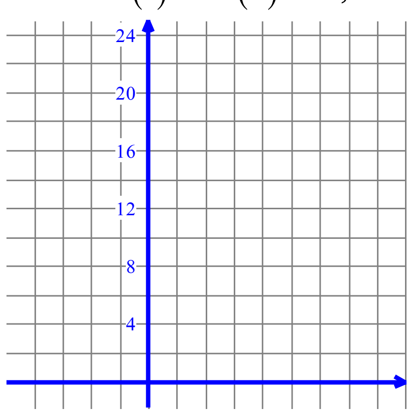

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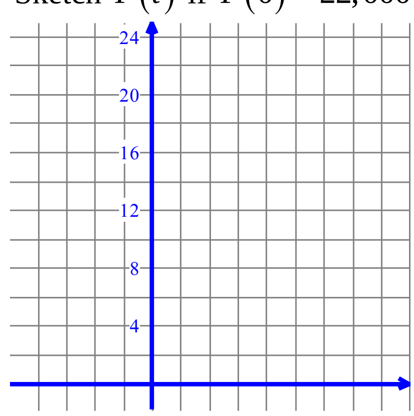

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