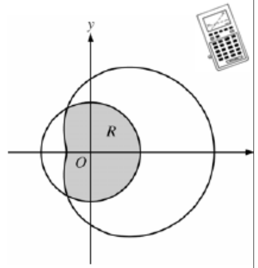
BC Calculus
10-#9

Name: ____________________________
Period: ____

1.



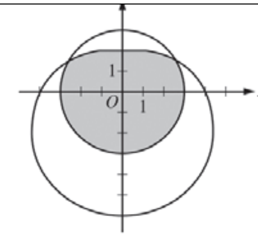
The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

(a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

(b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

2.



The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

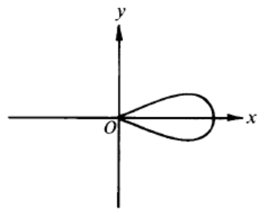
(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

3. The area of the closed region bounded by the polar graph of $r=\sqrt{3+\cos\theta}$ is given by the integral

(A) $\int_0^{2\pi}\sqrt{3+\cos\theta}\,d\theta$ (B) $\int_0^{\pi}\sqrt{3+\cos\theta}\,d\theta$ (C) $2\int_0^{\pi/2}(3+\cos\theta)\,d\theta$

(D) $\int_0^{\pi}(3+\cos\theta)\,d\theta$ (E) $2\int_0^{\pi/2}\sqrt{3+\cos\theta}\,d\theta$

4.



Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve $r=4\cos(3\theta)$ shown in the figure above?

(A) $16\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\cos(3\theta)\,d\theta$ (B) $8\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos(3\theta)\,d\theta$ (C) $8\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\cos^2(3\theta)\,d\theta$

(D) $16\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2(3\theta)\,d\theta$ (E) $8\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2(3\theta)\,d\theta$

5. The area of the region inside the polar curve $r=4\sin\theta$ and outside the polar curve $r=2$ is given by

(A) $\frac{1}{2}\int_0^{\pi}(4\sin\theta-2)^2\,d\theta$ (B) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}(4\sin\theta-2)^2\,d\theta$ (C) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(4\sin\theta-2)^2\,d\theta$

(D) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(16\sin^2\theta-4)\,d\theta$ (E) $\frac{1}{2}\int_0^{\pi}(16\sin^2\theta-4)\,d\theta$

6. If $x=t^3-t$ and $y=\sqrt{3t+1}$, then $\frac{dy}{dx}$ at $t=1$ is

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{3}{4}$ (D) $\frac{8}{3}$ (E) 8

7. A curve in the plane is defined parametrically by the equations $x=t^3+t$ and $y=t^4+2t^2$.
An equation of the line tangent to the curve at $t=1$ is

(A) $y=2x$ (B) $y=8x$ (C) $y=2x-1$

(D) $y=4x-5$ (E) $y=8x+13$

8. A particle moves in the xy-plane so that at any time t its coordinates are $x=t^2-1$ and $y=t^4-2t^3$.
At $t=1$, its acceleration vector is

(A) $(0,-1)$ (B) $(0,12)$ (C) $(2,-2)$ (D) $(2,0)$ (E) $(2,8)$

9. For time $t\geq 0$ seconds, the position of an object traveling along a curve in the xy-plane is given by the parametric equations $x(t)$ and $y(t)$, where $\frac{dx}{dt}=t^2+3$ and $\frac{dy}{dt}=t^3+t$.
At what time t is the speed of the object 10 units per second?

(A) 1.675

(B) 1.813

(C) 4.217

(D) 10.191

10. The asymptotes of the graph of the parametric equations $x=\frac{1}{t}$, $y=\frac{t}{t+1}$ are

(A) $x=0,\ y=0$ (B) $x=0$ only (C) $x=-1,\ y=0$

(D) $x=-1$ only (E) $x=0,\ y=1$

11. A particle moves on the curve $y=\ln x$ so that the x-component has velocity $x'(t)=t+1$ for $t\geq 0$.
At time $t=0$, the particle is at the point $(1,0)$. At time $t=1$, the particle is at the point

(A) $(2,\ln 2)$ (B) $(e^2,2)$ (C) $\left(\frac{5}{2},\ln\frac{5}{2}\right)$

(D) $(3,\ln 3)$ (E) $\left(\frac{3}{2},\ln\frac{3}{2}\right)$

12. A curve is defined by the parametric equations $x(t)=3e^{2t}$ and $y(t)=e^{3t}-1$.
What is $\frac{d^2y}{dx^2}$ in terms of t?

(A) $\frac{1}{12e^t}$ (C) $\frac{e^t}{2}$

(B) $\frac{1}{9e^t}$ (D) $\frac{3e^t}{4}$

13. The length of the curve determined by the equations $x=t^2$ and $y=t$ from $t=0$ to $t=4$ is

(A) $\int_0^4\sqrt{4t+1}\,dt$ (C) $\int_0^4\sqrt{2t^2+1}\,dt$ (E) $2\pi\int_0^4\sqrt{4t^2+1}\,dt$

(B) $2\int_0^4\sqrt{t^2+1}\,dt$ (D) $\int_0^4\sqrt{4t^2+1}\,dt$

14. For what values of t does the curve given by the parametric equations $x=t^3-t^2-1$ and $y=t^4+2t^2-8t$ have a vertical tangent?

(A) 0 only (C) 0 and $\frac{2}{3}$ only (D) 0, $\frac{2}{3}$, and 1

(B) 1 only (E) No value

15. In the xy-plane, the graph of the parametric equations $x=5t+2$ and $y=3t$, for $-3\leq t\leq 3$, is a line segment with slope

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13