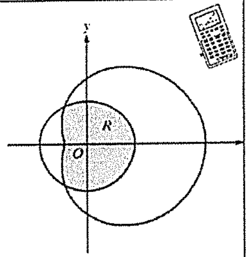
BC Calculus
11-#7

Name: Key
Period: ____

1. The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.



(a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

(b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$.

Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

a)
$$\text{Area} = \frac{4\pi/3}{2}(2)^2 + \frac{1}{2}\int_{2\pi/3}^{4\pi/3}(3+2\cos\theta)^2\,d\theta$$
$$= \boxed{10.370}$$

b) $\left.\frac{dr}{dt}\right|_{\theta=\pi/3} = \left.\frac{dr}{d\theta}\right|_{\theta=\pi/3} = -1.732$

The particle is getting closer to the origin at $\theta = \pi/3$ since $\frac{dr}{d\theta} < 0$.

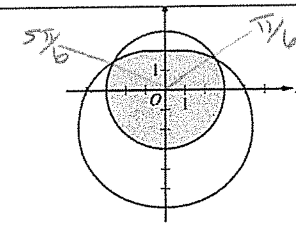
c) $y = r\sin\theta = (3+2\cos\theta)\sin\theta$

$\left.\frac{dy}{dt}\right|_{\theta=\pi/3} = \left.\frac{dy}{d\theta}\right|_{\theta=\pi/3} = 0.5$

$y(\pi/3) = (3 + 1/2)(\sqrt{3}/2) > 0$

The particle is moving away from the x-axis since $dy/dt > 0$ and $y > 0$ when $\theta = \pi/3$

2.



area of sector
$\frac{\theta}{2} r^2$

The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

a)
$$\text{Area} = \frac{4\pi/3}{2}(3)^2 + \frac{1}{2}\int_{\pi/6}^{5\pi/6}(4-2\sin\theta)^2\,d\theta$$
$$= \boxed{24.709}$$

b) $r = 4 - 2\sin(\theta)$

$x(\theta) = (4 - 2\sin\theta)\cos\theta$

$x(t) = (4 - 2\sin(t^2))\cos(t^2)$

$x(t) = -1$

$\boxed{t = 1.428}$

c) Position vector:

$\langle (4-2\sin(t^2))\cos(t^2), (4-2\sin(t^2))\sin(t^2) \rangle$

$v(1.5) = \langle x'(1.5), y'(1.5) \rangle$

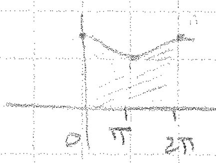
$= \langle -8.072, -1.673 \rangle$

3. $r = \sqrt{3+\cos\theta}$

$\frac{1}{2}\int_0^{2\pi} \left(\sqrt{3+\cos\theta}\right)^2 d\theta$

$\frac{1}{2}\int_0^{2\pi} (3+\cos\theta)\, d\theta$

$\int_0^{\pi} (3+\cos\theta)\, d\theta$



$\boxed{D}$

4. $r = 4\cos(3\theta)$

$4\cos(3\theta) = 0$

$\cos(3\theta) = 0$

$3\theta = \pi/2,\ 3\pi/2 \ldots$

$\theta = \pi/6,\ \pi/2 \ldots$

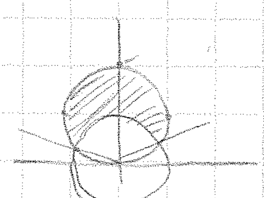
$\frac{1}{2}\int_{-\pi/6}^{\pi/6} (4\cos(3\theta))^2\, d\theta$

$\frac{1}{2}\cdot 16 \int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\, d\theta$

$\boxed{E}$

5. $r = 4\sin\theta$

$r = 2$



$\theta = \pi/6$

$\frac{1}{2}\int_{\pi/6}^{5\pi/6} (4\sin\theta)^2 - \frac{1}{2}\int_{\pi/6}^{5\pi/6} 2^2\, d\theta$

$\frac{1}{2}\int_{\pi/6}^{5\pi/6} 16\sin^2\theta - 4\, d\theta$ $\boxed{D}$

6. $x = t^3 - t$

$y = \sqrt{3t+1}$

$\frac{dy}{dx} = \frac{\frac{1}{2}(3t+1)^{-1/2}\cdot 3}{3t^2-1}$ $\boxed{B}$

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{\frac{1}{2}\left(\frac{1}{\sqrt{4}}\right)\cdot 3}{3-1} = \frac{3/4}{2} = \boxed{3/8}$

7. $x = t^3 + t$

$y = t^4 + 2t^2$

$\frac{dy}{dx} = \frac{4t^3+4t}{3t^2+1}$ $\left.\frac{dy}{dx}\right|_{t=1} = 8/4 = 2$

$x(1) = 2$

$y(1) = 3$

$y - 3 = 2(x-2)$

$y = 2x - 4 + 3$

$y = 2x - 1$ $\boxed{C}$

8. $x = t^2 - 1$

$y = t^4 - 2t^3$

$V(t) = \langle 2t,\ 4t^3 - 6t^2 \rangle$

$V'(t) = a(t) = \langle 2,\ 12t^2 - 12t \rangle$

$V'(1) = a(1) = \langle 2, 0 \rangle$ $\boxed{D}$

9. speed: $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 10$

$\sqrt{(t^2+3)^2 + (t^3+t)^2} = 10$

$t = 1.813$ $\boxed{B}$

10. $x = \frac{1}{t} \to t = 1/x$

$y = \frac{t}{t+1} \to y = \frac{1/x}{1/x+1} \to y = \frac{1}{1+x}$

VA: $1 + x = 0$

$\boxed{x = -1}$ $\boxed{C}$

HA: $\lim_{x\to\infty} \frac{1}{1+x} = 0$ $\boxed{y=0}$

11. $x(1) = x(0) + \int_0^1 t+1\, dt$

$= 1 + \left[\frac{1}{2}t^2 + t\right]_0^1$

$= 1 + \frac{1}{2} + 1 - 0 = 5/2$

$(5/2,\ \ln(5/2))$ $\boxed{C}$

12. $x(t) = 3e^{2t}$

$y(t) = e^{3t} - 1$

$$\frac{dy}{dx} = \frac{e^{3t} \cdot \cancel{3}}{\cancel{3}e^{2t} \cdot 2}$$

$$\frac{dy}{dx} = \frac{e^{t}}{2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{1}{2}e^{t}}{6e^{2t}}$$

$$= \frac{1}{12e^{t}} \quad \boxed{A}$$

13. $x = t^2 \quad dx/dt = 2t$

$y = t \quad dy/dt = 1$

$$\int_0^4 \sqrt{(2t)^2 + 1^2}\, dt$$

$$\int_0^4 \sqrt{4t^2 + 1}\, dt \quad \boxed{D}$$

14. $x = t^3 - t^2 - 1$

$y = t^4 + 2t^2 - 8t$

$$\frac{dx}{dt} = 3t^2 - 2t = 0$$

$3t^2 - 2t = 0$

$t(3t - 2) = 0$

$t = 0 \quad 3t = 2$

$t = 2/3$

$$\frac{dy}{dt} = 4t^3 + 4t - 8$$

$\boxed{D}$

make sure

$\frac{dy}{dt} \neq 0$ at

$t = 0$ or $t = 2/3$

15. $x = 5t + 2$

$y = 3t$

$t = y/3$

$x = 5(y/3) + 2$

$3x = 5y + 6$

$-5y = -3x + 6$

$y = \frac{3}{5}x - \frac{6}{5} \quad m = \frac{3}{5}$

$\boxed{A}$

3. D
4. E
5. D
6. B
7. C
8. D
9. B
10. C
11. C
12. A
13. D
14. D
15. A