3-47 Continued p. 225: 33, 34, 35, 36, 39, 40, 41 2018

33) $4x + y = 108$

$y = 108 - 4x$

$V = x^2 y$

$V(x) = x^2(108 - 4x)$

$= 108x^2 - 4x^3$

$V'(x) = 216x - 12x^2$

$= 12x(18 - x) = 0$

$x = 0 \quad x = 18$

max

$y = 108 - 4(18)$

$= 32$

$\boxed{\text{Dimensions } 18 \times 18 \times 32 \text{ for box with max volume}}$

34. $2\pi r + y = 108$

$y = 108 - 2\pi r$

$V = \pi r^2 y$

$V(r) = \pi r^2 (108 - 2\pi r)$

$= 2\pi(54r^2 - \pi r^3)$

$= 2\pi(108r - 3\pi r^2)$

$= 6\pi r(36 - \pi r) = 0$

$r = 0 \quad r = 36/\pi$

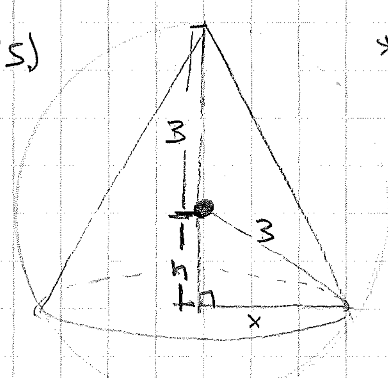
$y = 108 - 2\pi(36/\pi)$

$= 108 - 72$

$= 36$

$\boxed{\text{Dimensions } r = 36/\pi, \ y = 36}$

35)



* use 3 for radius of sphere.

35) $V = 1/3\, \pi x^2 (3 + h)$

Constraint: $x^2 + h^2 = 9$

$x^2 = 9 - h^2$

$V(h) = \pi/3 (9 - h^2)(3 + h)$

$= \pi/3 (27 + 9h - 3h^2 - h^3)$

$V'(h) = \pi/3 (0 + 9 - 6h - 3h^2)$

$= -3\pi/3 (h^2 + 2h - 3)$

$= -\pi(h + 3)(h - 1)$

$h = -3 \quad \boxed{h = 1}$

$x^2 = 9 - 1^2$

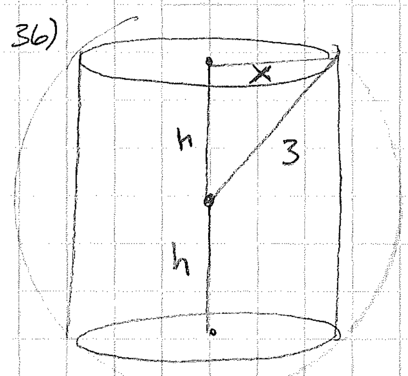
$x^2 = 8$

$V = \pi/3 (8)(3 + 1)$

$= \pi/3 (8)(4)$

$= \frac{32\pi}{3}$

36)



* use 3 for radius of sphere

$x^2 + h^2 = 9$

$x^2 = 9 - h^2$

$V = \pi x^2 2h$

$V(h) = \pi(9 - h^2)2h$

$= 2\pi(9h - h^3)$

$V'(h) = 2\pi(9 - 3h^2)$

$= 6\pi(3 - h^2) = 0$

$h = \sqrt{3}$

$x^2 = 9 - (\sqrt{3})^2$

$x^2 = 6$

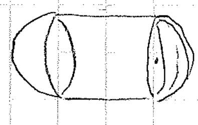
$x = \sqrt{6}$

Dimensions: Cylinder's

radius: $x = \sqrt{3}$ height: $2\sqrt{6}$

Volume: $\pi \cdot 6 \cdot 2\sqrt{3} = 12\pi\sqrt{3}$

39)



$V = 12$

$V = \frac{4}{3}\pi r^3 + \pi r^2 h = 12$

$\pi r^2 h = 12 - \frac{4}{3}\pi r^3$

$h = \frac{12 - \frac{4}{3}\pi r^3}{\pi r^2}$

$h = \frac{12}{\pi r^2} - \frac{4}{3} r$

$\text{Area} = 4\pi r^2 + 2\pi r h$

$A(r) = 4\pi r^2 + 2\pi r\left(\frac{12}{\pi r^2} - \frac{4}{3} r\right)$

$= \frac{4\pi r^2}{1} + \frac{24}{r} - \frac{8\pi}{3} r^2$

$= \frac{4\pi}{1} r^2 - \frac{8\pi}{3} r^2 + 24 r^{-1}$

$= \frac{12\pi}{3} r^2 - \frac{8\pi}{3} r^2 + 24 r^{-1}$

$= \frac{4\pi}{3} r^2 + 24 r^{-1}$

$A'(r) = \frac{8\pi}{3} r - \frac{24}{r^2}$

$= \frac{8\pi r^3 - 72}{3r^2}$

$A'(r) = 0$

$8\pi r^3 = 72$

$r^3 = 9/\pi$

$\boxed{r = \sqrt[3]{9/\pi}}$

40) $V = 3000$

$\frac{4}{3}\pi r^3 + \pi r^2 h = 3000$

$\pi r^2 h = 3000 - \frac{4}{3}\pi r^3$

$h = \frac{3000}{\pi r^2} - \frac{4\pi r^3}{3\pi r^2}$

$h = \frac{3000}{\pi r^2} - \frac{4}{3} r$

$\text{Cost} = 2(4\pi r^2) + 1(2\pi r h)$

$C(r) = 8\pi r^2 + 2\pi r\left(\frac{3000}{\pi r^2} - \frac{4}{3} r\right)$

$= \frac{8\pi r^2}{1} + 6000 r^{-1} - \frac{8\pi}{3} r^2$

$= \frac{16\pi}{3} r^2 + 6000 r^{-1}$

$C'(r) = \frac{32\pi}{3} r - \frac{6000}{r^2}$

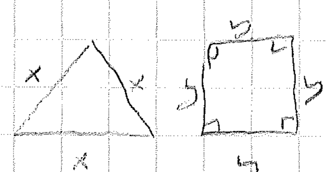
$= \frac{32\pi r^3 - 18000}{3r^2}$

$C'(r) = 0$

$32\pi r^3 = 18000$

$r = \sqrt[3]{\frac{1125}{2\pi}}$

41.



$3x + 4y = 10$

$4y = 10 - 3x$

$y = \frac{10}{4} - \frac{3}{4} x$

$A(x) = \frac{\sqrt{3}}{4} x^2 + y^2$

$= \frac{\sqrt{3}}{4} x^2 + \left(\frac{5}{2} - \frac{3}{4} x\right)^2$

$A'(x) = \frac{\sqrt{3}}{2} x + 2\left(\frac{5}{2} - \frac{3}{4} x\right) \cdot \frac{-3}{4}$

$= \frac{\sqrt{3}}{2} x + \frac{-3}{2}\left(\frac{5}{2} - \frac{3}{4} x\right)$

$= \frac{\sqrt{3}}{2} x - \frac{15}{4} + \frac{9}{8} x$

$A'(x) = 0 \quad \frac{9}{8} x + \frac{\sqrt{3}}{2} x = \frac{15}{4}$

$x\left(\frac{9 + 4\sqrt{3}}{8}\right) = \frac{15}{4} \rightarrow x = \frac{15 \cdot 8}{4(9 + 4\sqrt{3})}$

$x = \frac{30}{9 + 4\sqrt{3}}$

$\boxed{x = \frac{30}{9 + 4\sqrt{3}}}$ side of $\triangle$

$y = \frac{10}{4} - \frac{3}{4}\left(\frac{30}{9 + 4\sqrt{3}}\right)$

side of $\square$.

$y = \frac{10(9 + 4\sqrt{3}) - 90}{4(9 + 4\sqrt{3})} = \boxed{\frac{10\sqrt{3}}{4\sqrt{3} + 9}}$