4-#11 Solutions

1. $\int \sin(2x+3)\,dx$

$-\cos(2x+3)\cdot \frac{1}{2} + C$ **D**

2. $\int_0^8 \frac{dx}{\sqrt{1+x}}$

$\left[2(1+x)^{1/2}\right]_0^8$

$2\left[\sqrt{9} + \sqrt{1}\right]$

$2[3-1]$

$2(2) = 4$ **D**

3. $h(x) = f^2(x) - g^2(x)$

$h'(x) = 2f(x)\cdot f'(x) - 2g(x)\cdot g'(x)$

$f'(x) = -g(x)$

$g'(x) = f(x)$

$h'(x) = 2f(x)\cdot -g(x) - 2g(x)\cdot f(x)$

$= -4f(x)g(x)$ **C**

4. $F(x) = \int_0^x e^{-t^2}\,dt$

$F'(x) = e^{-x^2}$ **E**

5. $\frac{dy}{dx} = \frac{-x}{y e^{x^2}}$

$\int y\,dy = \int \frac{-x}{e^{x^2}}\,dx$

$\frac{1}{2}y^2 = \int -x e^{-x^2}\,dx \quad u = -x^2$

$\frac{1}{2}y^2 = \frac{1}{2}\int e^u\,du \quad du = -2x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}e^u + C \quad \frac{1}{2}du = -x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}e^{-x^2} + C$

$y^2 = e^{-x^2} + C \quad (0,2)$

$4 = 1 + C$

$3 = C$

$y^2 = e^{-x^2} + 3$

$y = \sqrt{e^{-x^2}+3}$ **D**

6. $\frac{1}{2-0}\int_0^2 x^2\sqrt{x^3+1}\,dx \quad u = x^3+1, \; du = 3x^2\,dx$

$\frac{1}{2}\cdot\frac{1}{3}\int_1^9 \sqrt{u}\,du$

$\frac{1}{6}\left[\frac{2}{3}u^{3/2}\right]_1^9$

$\frac{1}{6}\cdot\frac{2}{3}\left[27 - 1\right] = \frac{1}{9}(26)$ **A**

7. $\int_0^{\pi/4} \tan^2 x\,dx$

$\int_0^{\pi/4} (\sec^2 x - 1)\,dx$

$[\tan x - x]_0^{\pi/4}$

$(1 - \pi/4) - (0 - 0)$

$1 - \pi/4$ **B**

8. $\frac{dA}{dt} = 96\pi \text{ m}^2/\text{sec}$

Find $\frac{dr}{dt}$ when $A = 64\pi \text{ m}^2$

$A = \pi r^2 \qquad r = 8$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt}\cdot\frac{1}{2\pi r} = \frac{dr}{dt}$

$\frac{96\pi}{2\pi\cdot 8} = \frac{dr}{dt}$

$6 = dr/dt$ **A**

9. $\frac{1}{2}(4+k)(2)$

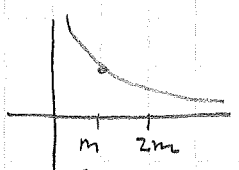
$\frac{1}{2}(k+8)(2)$

$\frac{1}{2}(8+12)(2)$

$4 + 2k + 16 + 12 = 52$

$2k = 52 - 32$

$k = 10$ **D**

10. 

$\int_m^{2m} 1/x\,dx$ **A**

$[\ln(x)]_m^{2m}$

$\ln(2m) - \ln(m) = \ln\left(\frac{2m}{m}\right)$

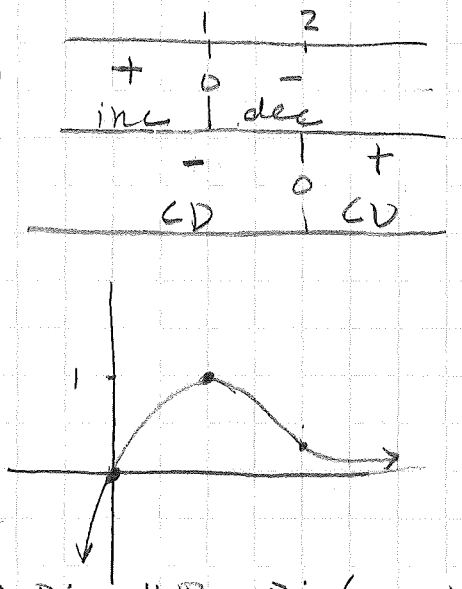
$= \ln(2)$

11. $f(x) = xe^{1-x}$

a) $f(0) = 0 \cdot e^1$
$= 0$
$(0, 0)$ only

b) $f'(x) = x \cdot e^{1-x} \cdot -1 + e^{1-x}$
$= e^{1-x}(-x+1)$
$= e^{1-x}(1-x)$
$f''(x) = e^{1-x}(-1) + (1-x)e^{1-x} \cdot -1$
$= e^{1-x}(-1 + (-1)(1-x))$
$= e^{1-x}(-1-1+x)$
$= e^{1-x}(x-2)$

c)



d) D: all $\mathbb{R}$ R: $(-\infty, 1]$

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12. $-8x^2 + 5xy + y^3 = -149$

a) $-16x + 5(xy' + 1 \cdot y) + 3y^2 y' = 0$
$5xy' + 3y^2 y' = 16x - 5y$
$y' = \frac{16x - 5y}{5x + 3y^2}$

b) $y'|_{(4,-1)} = \frac{64+5}{20+3} = \frac{69}{23} = 3$

$y + 1 = 3(x - 4)$
$y = 3x - 12 - 1$
$y = 3x - 13$

c) $k \approx 3(4.2) - 13$
$12.6 - 13 = \boxed{-.4}$

d) $-8(4.2)^2 + 5(4.2)k + k^3 = -149$
$k = -.373$

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13. $v(t) = \ln(t^2 - 3t + 3)$ $\quad x(0) = 8$

a) $a(t) = \frac{1}{t^2 - 3t + 3} \cdot (2t - 3)$
$a(4) = 0.714$

b) $v(t) = 0$
$t = 1$ and $t = 2$
travel left if $v(t) < 0$
$1 < t < 2$

c) $x(2) = x(0) + \int_0^2 v(t)\,dt$
$= 8 + .369$
$= 8.369$

d) $\frac{1}{2}\int_0^2 |v(t)|\,dt$

$= .371$