BC Calculus
7-#3 Handout (there is book work too)

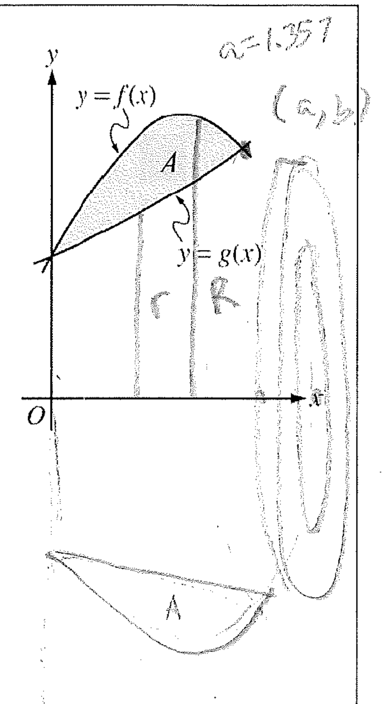
Name: ____________________________

1.

Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let A be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

The region A is revolved around the x-axis.

a) Sketch A reflected over the x-axis.
b) Draw a typical slice.
c) Draw vertical lines for R and r on the graph.
d) Write an integral to find the volume and use your calculator to solve it.



$R = f(x) = 1+\sin(2x)$

$r = g(x) = e^{x/2}$

$V = \pi \int_0^a f^2(x) - g^2(x)\,dx = \boxed{4.267}$

2.

If $x+7y=29$ is an equation of the line normal to the graph of f at the point $(1,4)$, then $f'(1) =$

(A) 7 (B) $\frac{1}{7}$ (C) $-\frac{1}{7}$ (D) $-\frac{7}{29}$ (E) -7

$7y = -x+29$

$y = -\frac{1}{7}x + \frac{29}{7}$

3.

Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$

II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

$F'(x) = \frac{1}{2} \cdot 2\sin x \cdot \cos x$
$= \sin x \cos x$

$F'(x) = \dfrac{\sin(2x)\cdot 2}{4}$
$= \dfrac{2\sin x \cos x \cdot 2}{4}$

4.

If the graph of $y = \dfrac{ax+b}{x+c}$ has a horizontal asymptote $y=2$ and a vertical asymptote $x=-3$, then $a+c =$

$\lim_{x\to\infty} \dfrac{ax+b}{x+c} = a = 2$ $c = 3$

(A) -5 (B) -1 (C) 0 (D) 1 (E) 5

5.

$\lim_{\theta\to 0} \dfrac{1-\cos\theta}{2\sin^2\theta}$ is $= \lim_{\theta\to 0} \dfrac{\sin\theta}{4\sin\theta\cdot\cos\theta} = \dfrac{1}{4(1)} = \frac{1}{4}$

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent

6. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b.

(b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k?

(a)

$f'(-1)=0$

$f''(-2)=0$

$f'(x)=12x^2+2ax+b$

$f''(x)=24x+2a$

$f''(-2)=24(-2)+2a=0$

$-48+2a=0$

$2a=48$

$\boxed{a=24}$

$f'(x)=12x^2+48x+b$

$f'(-1)=12-48+b=0$

$=-36+b=0$

$\boxed{b=36}$

(b)

$\int_0^1 4x^3+24x^2+36x+k\,dx=32$

$\left[x^4+8x^3+18x^2+kx\right]_0^1=32$

$1+8+18+k=32$

$27+k=32$

$\boxed{k=5}$

7. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6-2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6-2x)$ with the initial condition $f(3) = \frac{1}{4}$.

(a)

$\frac{dy}{dx}=y^2(6-2x)\Big|_{(3,1/4)}=\frac{1}{16}(0)=0$

$\frac{d^2y}{dx^2}=y^2\cdot(-2)+(6-2x)\cdot 2yy'$

$=-2y^2+(6-2x)2yy'$

$\frac{d^2y}{dx^2}\Big|_{(3,1/4)}=-2(1/16)+0$

$=\boxed{-1/8}$

(b)

$\int\frac{1}{y^2}\,dy=\int 6-2x\,dx$

$-y^{-1}=6x-x^2+C$

$y^{-1}=x^2-6x+C$

$y=\frac{1}{x^2-6x+C}$

$1/4=\frac{1}{9-18+C}$

$\frac{1}{4}=\frac{1}{-9+C}$

$C-9=4$

$C=13$

$\boxed{y=\frac{1}{x^2-6x+13}}$