7-#5:

1. a) $\int_9^{17} E(t)\,dt = 6004$ people

b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = \$104{,}048$

c) $H(t) = \int_9^t E(x) - L(x)\,dx$ $\qquad H(17) = 3725$

$H'(t) = E(t) - L(t)$

$H'(17) = E(17) - L(17) = -380$ people per hour

$H(17) = \int_9^{17} [E(x) - L(x)]\,dx = 3725$ people

At $t = 17$ (5:00 pm): there 3725 people in the park and the population in the park is decreasing by 380 people per hour.

d. $H'(t) = 0$
$E(t) - L(t) = 0$
$t = 15.795$

max people in the park at $t = 15.795$ hours

$+$ inc $\quad$ $-$ dec $\quad H'$
15.795

between 3:00 and 4:00 pm.

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2. $I(t) = 8$ $\qquad$ gallons/min
$L(t) = \sqrt{t+1}$ $\qquad$ $0 \le t \le 20$ minutes

$u = t+1$
$du = dt$

a. $\int_0^3 L(t)\,dt = 14/3$ gallons

$\int_0^3 \sqrt{t+1}\,dt = \int_1^4 u^{1/2}\,du$
$= \left[\tfrac{2}{3}u^{3/2}\right]_1^4$
$= 16/3 - \tfrac{2}{3} = 14/3$

b. $A(t)$ = water in the tank at time t.

$A(t) = 30 + \int_0^t I(t) - L(t)\,dt$

$A(3) = 30 + \int_0^3 I(t) - L(t)\,dt = 148/3$ gallons

c. $A(t) = 30 + \int_0^t 8 - L(x)\,dx$

d. $A'(t) = 8 - L(t)$
$8 - L(t) = 0$
$t = 63$

$w(0) = 30$
$w(63) = 163.\overline{3}$
$w(120) = 73.\overline{3}$

max at $t = 63$. since an analysis of all critical values and end points yields this maximum.

3. $P'(t) = 1 - 3e^{-.2\sqrt{t}}$ gallons/day

$P(0) = 50$

Safe when $P(t) \le 40$

a) $P'(9) = -.646$

Decreasing since $P'(9) < 0$

b) $P'(t) = 0$

$t = 30.173$

min at $t = 30.173$ since $P'(t) < 0$ for $t < 30.173$ and $p'(t) > 0$ for $t > 30.173$

c. $P(30.173)$

$= 50 + \int_0^{30.173} P'(t)\,dt$

$= 50 + (-14.89564)$

$= 35.104$

yes $P(30.173) < 40$

$35.104 < 40$

so safe.

d. $P(0) = 50$

$P'(0) = 1 - 3e^0 = -2$

$y = -2(x-0) + 50$

$y = -2x + 50$

$-2x + 50 = 40$

$-2x = -10$

$x = +5$

$t = 5$ lake safe according to tangent.

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4. $H(t) \to$ oil in

$R(t) \to$ oil out

a) $\int_0^{12} H(t)\,dt = 70.571$ gallons

b) $H(6) - R(6)$

$5.395 - 8.319 < 0$

Falling since $H(6) - R(6) < 0$

c) $125 + \int_0^{12} H(t) - R(t)\,dt$

$= 122.026$ gallons

d. Let $G(t) = 125 + \int_0^t H(x) - R(x)\,dx$ (volume in tank at time t).

$G'(t) = H(t) - R(t)$

$G'(t) = 0$

$t_1 = 4.790$ and $t_2 = 11.318$

$G(0) = 125$

$G(t_1) = 149.407$

$G(t_2) = 120.738$

$G(12) = 122.026$

$\boxed{\text{Min at } t = 11.318}$

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5. $F(t)$ traffic flow cars/min

a) $\int_0^{30} F(t)\,dt = 2474$ cars

b) $F'(7) = -1.873$ cars/min²

Dec since $F'(7) < 0$.

c) $\frac{1}{15-10} \int_{10}^{15} F(t)\,dt = 81.899$ cars/min

d) $\frac{F(15) - F(10)}{15 - 10} = 1.518$ cars/min/min

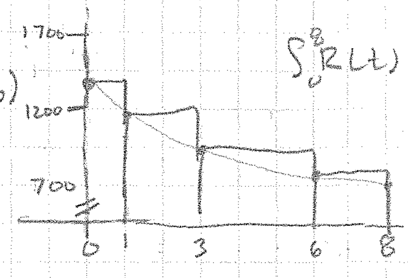
6. $w(t) = 2000e^{-t^2/20}$ liters / hour $\rightarrow$ <u>In</u>

$R(t)$ table $\rightarrow$ Out

a) $R'(2) = \dfrac{R(3) - R(1)}{3-1}$

$= -120$ liters/hr^2

b) $\int_0^8 R(t)\,dt \approx$



$\int_0^8 R(t)\,dt \approx$

$1 \cdot R(0) + 2R(1) + 3R(3) + 2R(6)$

$= 8050$ Liters

over estimate since $R(t)$ is decreasing.

c) Let $L(t)$ be the amount of water in tank at time t.

$L(t) = 50000 + \int_0^t w(x) - R(x)\,dx$

$L(8) \approx 50000 + \int_0^8 w(x)\,dt - \int_0^8 R(x)\,dx$

$\approx 50{,}000 + 7836.195 - 8440$

$\approx 49{,}396$ liters

d) yes. If $w(t) = R(t)$ then $w(t) - R(t) = 0$ for some t on $[0, 8]$. Since $w(0) - R(0) > 0$ and $w(8) - R(8) < 0$ the IVT guarantees that there exists some t such that $w(t) - R(t) = 0$.

7a) $k(x) = f(g(x))$

$k(3) = f(g(3))$
$= f(6)$
$= 4$

$k'(x) = f'(g(x)) \cdot g'(x)$
$k'(3) = f'(g(3)) \cdot g'(3)$
$= f'(6) \cdot 2$
$= 5 \cdot 2$
$= 10$

$y - 4 = 10(x-3)$

b) $h(x) = \dfrac{g(x)}{f(x)}$

$h'(x) = \dfrac{f(x) \cdot g'(x) - g(x) \cdot f'(x)}{f^2(x)}$

$h'(1) = \dfrac{f(1) \cdot g'(1) - g(1) \cdot f'(1)}{f^2(1)}$

$= \dfrac{-6 \cdot 8 - 2 \cdot 3}{(-6)^2} = \dfrac{-54}{36} = \dfrac{-3}{2}$

c. $\int_1^3 f''(2x)\,dx = \left[\tfrac{1}{2} f'(2x)\right]_1^3$

$= \tfrac{1}{2}\left[f'(6) - f'(2)\right]$

$\tfrac{1}{2}\left[5 - (-2)\right] = \tfrac{7}{2}$

8. $y^3 - xy = 2 \qquad \dfrac{dy}{dx} = \dfrac{y}{3y^2 - x}$

a) $\left.\dfrac{dy}{dx}\right|_{(-1,1)} = \dfrac{1}{3 - (-1)} = \dfrac{1}{4}$

$y = \tfrac{1}{4}(x+1) + 1$

b) $3y^2 - x = 0$
$3y^2 = x$
$y^3 - 3y^2 \cdot y = 2$
$y^3 - 3y^3 = 2$
$-2y^3 = 2$
$y^3 = -1$
$y = -1$

$(-1)^3 - x(-1) = 2$
$-1 + x = 2$
$x = 3$
$(3, -1)$

c) $\dfrac{d^2y}{dx^2} = \dfrac{(3y^2 - x) \cdot \frac{dy}{dx} - y \cdot \left[6y \cdot \frac{dy}{dx} - 1\right]}{(3y^2 - x)^2}$

$\left.(3y^2 - x)\right|_{(-1,1)} = 4$

$\left.\dfrac{d^2y}{dx^2}\right|_{(-1,1)} = \dfrac{4 \cdot \frac{1}{4} - 1\left[6 \cdot \frac{1}{4} - 1\right]}{4^2} = \dfrac{1 - \frac{1}{2}}{16} = \dfrac{\frac{1}{2}}{16} = \boxed{\dfrac{1}{32}}$