BC Calculus
9-#11 HW

Name: ______________________________
Period: _____

1. Use a Taylor series to evaluete the limit: $\lim_{x\to 0}\dfrac{\sin\left(x^3\right)-x^3}{x^9}$. Note: You could try to use L'Hopital's Rule but you would have to take many (how many?) derivatives before getting the answer.

2. Use an 8th degree polynomial to approximate $\int_{-1}^{1}\cos\left(x^2\right)dx$ without using a calculator. Use symmetry to make the problem a little easier. (The integrand should be degree 8, it should not have 8 terms.)

3. Let f be the function given by $f(x)=\ln\left(3-x\right)$. The third-degree Taylor polynomial for f about $x=2$ is

   (A) $-(x-2)+\dfrac{(x-2)^2}{2}-\dfrac{(x-2)^3}{3}$

   (B) $-(x-2)-\dfrac{(x-2)^2}{2}-\dfrac{(x-2)^3}{3}$

   (C) $(x-2)+(x-2)^2+(x-2)^3$

   (D) $(x-2)+\dfrac{(x-2)^2}{2}+\dfrac{(x-2)^3}{3}$

   (E) $(x-2)-\dfrac{(x-2)^2}{2}+\dfrac{(x-2)^3}{3}$

4. What is the approximation of the value of $\sin 1$ obtained by using the fifth-degree Taylor polynomial about $x=0$ for $\sin x$?

   (A) $1-\dfrac{1}{2}+\dfrac{1}{24}$

   (B) $1-\dfrac{1}{2}+\dfrac{1}{4}$

   (C) $1-\dfrac{1}{3}+\dfrac{1}{5}$

   (D) $1-\dfrac{1}{4}+\dfrac{1}{8}$

   (E) $1-\dfrac{1}{6}+\dfrac{1}{120}$

5. The Taylor series for $\sin x$ about $x=0$ is $x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\ldots$. If f is a function such that $f'(x)=\sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x=0$ is

   (A) $\dfrac{1}{7!}$ (B) $\dfrac{1}{7}$ (C) 0 (D) $-\dfrac{1}{42}$ (E) $-\dfrac{1}{7!}$

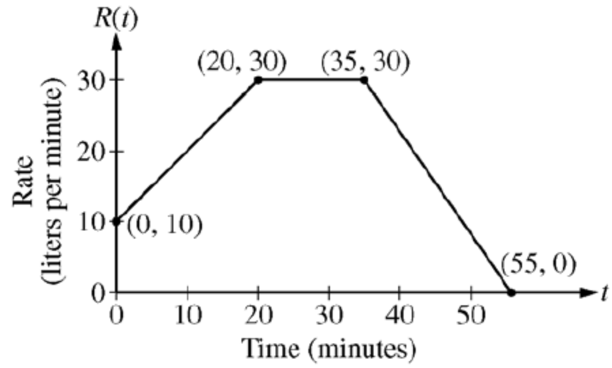
6. What are all values of x for which the series $\sum_{n=1}^{\infty}\dfrac{(x+2)^n}{\sqrt{n}}$ converges?

   (A) $-3<x<-1$ (B) $-3\le x<-1$ (C) $-3\le x\le -1$ (D) $-1\le x<1$ (E) $-1\le x\le 1$

7.	The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$? (A) $1+x^2+x^4+x^6+x^8+...$ (B) $x^2+x^3+x^4+x^5+...$ (C) $x^2+2x^3+3x^4+4x^5+...$ (D) $x^2+x^4+x^6+x^8+...$ (E) $x^2-x^4+x^6-x^8+...$
8.	A function has Maclaurin seris gvien by $\frac{x^4}{2!}+\frac{x^5}{3!}+\frac{x^6}{4!}+...+\frac{x^{n+3}}{(n+1)!}+....$ Hint: Factor out x^2 so that the power matches the factorial. Which of the following is an expression for $f(x)$? (A) $-3x\sin x+3x^2$ (B) $-\cos(x^2)+1$ (C) $-x^2\cos x+x^2$ (D) $x^2e^x-x^3-x^2$ (E) $e^{x^2}-x^2-1$
9.	What is the coefficient of x^2 in the Taylor series for $\frac{1}{(1+x)^2}$ about $x=0$? (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) 1 (D) 3 (E) 6
10.	Let $P(x)=3x^2-5x^3+7x^4+3x^5$ be the fifth-degree Taylor polynomial for the function *f* about $x=0$. What is the value of $f'''(0)$? (A) -30 (B) -15 (C) -5 (D) $\frac{-5}{6}$ (E) $\frac{-1}{6}$
11.	The coeffecient of $\left(x-\frac{\pi}{4}\right)^3$ in the Taylor series about $\frac{\pi}{4}$ of $f(x)=\cos(x)$ is (A) $\frac{\sqrt{3}}{12}$ (B) $\frac{-1}{12}$ (C) $\frac{1}{12}$ (D) $\frac{\sqrt{2}}{12}$ (E) $\frac{-\sqrt{2}}{6}$
12.	. If $f(x) = x\sin(2x)$, which of the following is the Taylor series for f about $x = 0$? (A) $x-\frac{x^3}{2!}+\frac{x^5}{4!}-\frac{x^7}{6!}+\cdots$ (B) $x-\frac{4x^3}{2!}+\frac{16x^5}{4!}-\frac{64x^7}{6!}+\cdots$ (C) $2x-\frac{8x^3}{3!}+\frac{32x^5}{5!}-\frac{128x^7}{7!}+\cdots$ (D) $2x^2-\frac{2x^4}{3!}+\frac{2x^6}{5!}-\frac{2x^8}{7!}+\cdots$ (E) $2x^2-\frac{8x^4}{3!}+\frac{32x^6}{5!}-\frac{128x^8}{7!}+\cdots$
13.	Let f be a function with $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$? (A) $2-(x-3)+3(x-3)^2+2(x-3)^3$ (B) $2-(x-3)+3(x-3)^2+4(x-3)^3$ (C) $2-(x-3)+6(x-3)^2+12(x-3)^3$ (D) $2-x+3x^2+2x^3$ (E) $2-x+6x^2+12x^3$

14.	
	At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.

At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.



(a) Find $R'(45)$. Using appropriate units, explain the meaning of your answer in the context of this problem.

(b) How many liters of water have been pumped into the tank from time $t = 0$ to time $t = 55$ minutes? Show the work that leads to your answer.

(c) At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D, where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

(d) Using the functions R and D, determine whether the amount of water in the tank is increasing or decreasing at time $t = 45$ minutes. Justify your answer.

15.

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24

Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \leq 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

(a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

(c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \dfrac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

(d) Write an expression for Nathan's acceleration in terms of t.