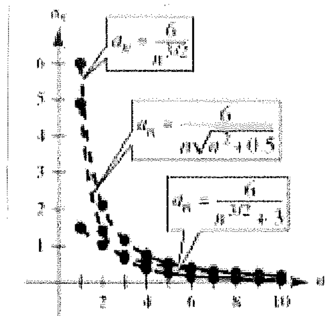
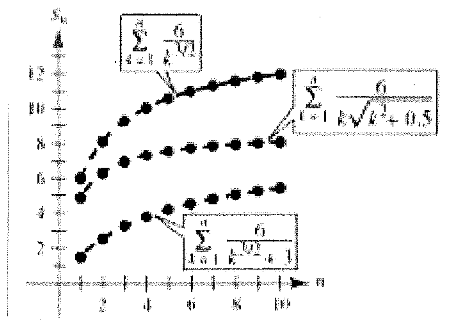
9-#4: 1a-c, 3, 5, 7, 15, 19, 29-36, 55, 59, 60

formally intuitively

1. a)





b) $\sum_{n=1}^{\infty} \frac{6}{n^{3/2}}$ p-series

Converges $p = 3/2 > 1$

c) The two series are "smaller" than the convergent p-series so they are both convergent. (Direct comparison test.)

3. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

I think it converges since $\sum 1/n^2$ converges

$0 < \frac{1}{n^2+1} \leq \frac{1}{n^2}$ for all n

Therefore $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges by comparison with convergent p-series $\sum_{2}^{\infty} 1/n^2$

5. $\sum_{n=2}^{\infty} \frac{1}{n-1}$

I think it diverges

$0 < \frac{1}{n} < \frac{1}{n-1}$ for $n \geq 2$

Therefore $\sum_{2}^{\infty} \frac{1}{n-1}$ diverges by comparison with divergent p-series $\sum_{2}^{\infty} 1/n$

7. $\sum_{n=0}^{\infty} \frac{1}{3^n+1}$

I think it converges $\sum (1/3)^n$

$0 < \frac{1}{3^n+1} < \frac{1}{3^n} = \left(\frac{1}{3}\right)^n$

Therefore $\sum_{n=0}^{\infty} \frac{1}{3^n+1}$ converges by comparison with convergent geometric series $\sum_{n=1}^{\infty} (1/3)^n$ with $|r| = 1/3 < 1$.

15. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

I think it diverges since $\sum 1/n$ diverges

$\lim_{n\to\infty} \frac{\frac{n}{n^2+1}}{1/n}$

$\lim_{n\to\infty} \frac{n}{n^2+1} \cdot \frac{n}{1} = 1 > 0$

therefore $\sum_{n=1}^{\infty} \frac{n}{n+1}$ diverges by a limit comparison with the divergent p-series $\sum_{n=1}^{\infty} 1/n$. $(p=1)$

19. $\sum_{n=1}^{\infty} \frac{2n^2-1}{3n^5+2n+1}$

I think it converges since $\sum 1/n^3$ converges

$\lim_{n\to\infty} \frac{\frac{2n^2-1}{3n^5+2n+1}}{1/n^3}$

$\lim_{n\to\infty} \frac{2n^2-1}{3n^5+2n+1} \cdot \frac{n^3}{1} = \frac{2}{3} > 0$

Therefore series converges by a limit comparison with convergent p-series $\sum 1/n^3$ $(p=3)$.

* These are not justified formally ... but rather intuitively. *

29. $\sum_{n=1}^{\infty} \sqrt{n}/n = \sum_{n=1}^{\infty} 1/\sqrt{n}$ $p = 1/2$:

p-series diverges

30. $\sum_{n=0}^{\infty} 5(-1/5)^n$

geometric $|r| = 1/5 < 1$
convergent

31. $\sum_{n=1}^{\infty} \frac{1}{3^n+2}$ convergent

compare to $\sum (1/3)^n$

32. $\sum_{n=4}^{\infty} \frac{1}{3n^2-2n-15}$

converges since $\sum_{n=4}^{\infty} 1/n^2$
converges

33. $\sum_{n=1}^{\infty} \frac{n}{2n+3}$ diverges

nth term
$\lim_{n\to\infty} \frac{n}{2n+1} = 1/2 \neq 0$

34. $\sum_{n=1}^{\infty} \frac{1}{n+1} - \frac{1}{n+2}$

$= \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} \ldots$

convergent telescoping
series

35. $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$ converges

since similar magnitude to
$\sum_{n=1}^{\infty} 1/n^3$

36. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$ converges since

similar magnitude to
$\sum_{n=1}^{\infty} 1/n^2$

55. False

59. True

60. False