9 - #7

1. $\dfrac{(n+1)!}{(n-2)!}$

$\dfrac{(n+1)(n)(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$

$(n+1)(n)(n-1)$ ✓

13. $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{3^n}$

$\displaystyle\lim_{n\to\infty} \frac{(n+1)!}{3^{n+1}} \cdot \frac{3^n}{n!}$

$\displaystyle\lim_{n\to\infty} \frac{(n+1)\,\cancel{n!}\cdot\cancel{3^n}}{\cancel{3^n}\cdot 3^1\cdot\cancel{n!}} = \infty$

Diverges by Ratio test

15. $\displaystyle\sum_{n=1}^{\infty} n(3/4)^n$

$\displaystyle\lim_{n\to\infty} \frac{(n+1)(3/4)^{n+1}}{n(3/4)^n}$

$\displaystyle\lim_{n\to\infty} \frac{(n+1)\cancel{(3/4)^n}\cdot 3/4}{n\,\cancel{(3/4)^n}}$

$= 3/4 < 1$

converges

17. $\displaystyle\sum_{n=1}^{\infty} n/2^n$

$\displaystyle\lim_{n\to\infty} \frac{n+1}{2^{n+1}} \cdot \frac{2^n}{n}$

$\displaystyle\lim_{n\to\infty} \frac{(n+1)(\cancel{2^n})}{\cancel{2^n}\cdot 2\cdot n} = \frac{1}{2} < 1$

converges

21. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\cdot 2^n}{n!}$

$\displaystyle\lim_{n\to\infty} \left| \frac{(-1)^{n+1}\,2^{n+1}}{(n+1)!} \cdot \frac{n!}{(-1)^n\cdot 2^n} \right|$

$\displaystyle\lim_{n\to\infty} \frac{\cancel{2^n}\cdot 2^1\cdot\cancel{n!}}{(n+1)\,\cancel{n!}\,\cancel{2^n}} = 0 < 1$

converges

23. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n\cdot 3^n}$

$\displaystyle\lim_{n\to\infty} \frac{(n+1)!}{(n+1)3^{n+1}} \cdot \frac{n\cdot 3^n}{n!}$

$\displaystyle\lim_{n\to\infty} \frac{\cancel{(n+1)}\,\cancel{n!}\;n\cdot\cancel{3^n}}{\cancel{(n+1)}\,\cancel{3^n}\cdot 3\;\cancel{n!}}$

$\displaystyle\lim_{n\to\infty} \frac{n}{3} = \infty$

diverges

33. $\displaystyle\sum_{n=1}^{\infty} 1/n^{3/2}$

$\displaystyle\lim_{n\to\infty} \frac{1}{(n+1)^{3/2}} \cdot \frac{n^{3/2}}{1}$

$\displaystyle\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{3/2}$

$\displaystyle\left(\lim_{n\to\infty} \frac{n}{n+1}\right)^{3/2} = 1^{3/2} = 1$

ratio test inconclusive
but converges by p-series
$p = 3/2 > 1$

51. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 5}{n}$

dec, $\lim_{n\to\infty} 5/n = 0$
converges A.S.T

52. Diverges
p-series $p=1$

53. $\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n}}$

$\sum_{n=1}^{\infty} 3/n^{3/2} = 3 \sum_{n=1}^{\infty} 1/n^{3/2}$

$\boxed{\text{converges}}$
p-series $p = 3/2$

54. $\sum_{n=1}^{\infty} (\pi/4)^n$

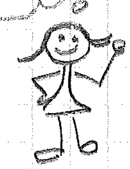
geometric $|r| = \pi/4 < 1$
converges

55. $\sum_{n=1}^{\infty} \frac{2n}{n+1}$

$\boxed{\text{diverges}}$
$\lim_{n\to\infty} \frac{2n}{n+1} \neq 0$
nth term test

56. $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ ratio will fail, try it

I think it diverges since similar to $\sum 1/n$.



Limit comparison with divergent $\sum 1/n$

$\lim_{n\to\infty} \frac{n}{2n^2+1} \cdot \frac{n}{1} = 1/2 > 0$

both $\boxed{\text{diverge}}$

57. $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 3^{n-2}}{2^n}$

$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} \cdot 3^{n+1-2}}{2^{n+1}} \cdot \frac{2^n}{(-1)^n \cdot 3^{n-2}} \right|$

$\lim_{n\to\infty} \frac{3^n \cdot 3^{-1} \cdot 2^n}{2^n \cdot 2 \cdot 3^n \cdot 3^{-2}} = \frac{3}{2} > 1$

$\boxed{\text{diverges}}$
by Ratio Test

61. $\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$ note $|\cos n| \leq 1$

I think it converges since $\sum 1/2^n$ converges
geometric

Direct Compare test, need to show that something larger converges

$\frac{\cos n}{2^n} \leq \frac{1}{2^n}$

and $\sum_{n=1}^{\infty} 1/2^n$ $\boxed{\text{convergent}}$
geometric, $|r| = 1/2 < 1$

70. B and C

73. $\sum_{n=1}^{\infty} (n/4^n) = \sum_{n=0}^{\infty} \frac{n+1}{4^{n+1}}$

p. 637:
43. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$

$|S - S_{10}| < |a_{11}| = \frac{1}{1331}$

If S_{10} is used to approximate the sum of the series then the error will be less than $\frac{1}{1331}$.

44. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

$|S - S_{31}| < |a_{32}| = \frac{1}{1024}$

If S_{31} is used to appox the sum of series then the error is less than $a_{32} = 1/1024$

31 terms

45. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3 - 1}$

$2n^3 - 1 = 1000$
$2n^3 = 1001$
$n^3 = 500.5$
$n = 7.94$

$|S - S_7| < |a_8| = 1/1023$

If S_7 is used to approx the sum of the series then the error $< a_8 = 1/1023$

7 terms

46. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$

$|S - S_5| < |a_6| = \frac{1}{1296}$

If S_5 is used to approximate the sum of the series then the error $< a_6 = 1/1296$

5 terms