BC Calculus — Name: ______________________________

9-#9: Error in Taylor Polynomials — Period: _____

General Form of a Taylor Polynomial for $f(x)$

$$P_n(x) = \frac{f(c)(x-c)^0}{0!} + \frac{f'(c)(x-c)^1}{1!} + \frac{f''(c)(x-c)^2}{2!} + \ldots + \frac{f^{(n)}(c)(x-c)^n}{n!}$$

Ex 1: Find the 4th degree Taylor Polynomial for $f(x) = \cos(x)$ centered at $x = \pi$.

Error Bound: If $P_n(x_0)$ is used to approximate $f(x_0)$ then $P_n(x_0) - Error \le f(x_0) \le P_n(x_0) + Error$

Alternating Series Error Bound:

If $P_n(x)$ is **alternating** then the

$$Error \le \left| \frac{f^{(n+1)}(c)(x_0 - c)^{n+1}}{(n+1)!} \right|$$

or "the error is less than the first omitted term."

***Lagrange* Error Bound:**

If $P_n(x)$ is **non-alternating*** then the

***Note: Lagrange can be used for alternating polynomials, but the other bound is easier and usually more accurate.**

$$Error \le \frac{\max\left|f^{(n+1)}(z)\right| \left|x_0 - c\right|^{n+1}}{(n+1)!}$$

where *z* is between x_0 and *c*.

Ex 2: Let *f* be a function having derivatives for all orders for all real numbers. The 3rd degree Taylor polynomial for *f* about $x = 2$ is given by $P_3(x) = 6 + 4(x-2) - \frac{7}{2}(x-2)^2 + \frac{4}{3}(x-2)^3$.

a) Use $P_3(x)$ to approximate $f(1.7)$.

b) Suppose the 4th derivative of *f* satisfies the inequality $\left|f^{(4)}(x)\right| \le 9$ for all *x* on the closed interval [1.7,2]. What is the Lagrange error bound for the maximum error of $f(1.7)$.

c) Use the Lagrange error bound on the approximation of $f(1.7)$ to find an interval $[a,b]$ such that $a \le f(1.7) \le b$

Ex 3: $\cos(x) \approx 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$

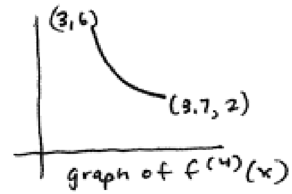
a) Approximate $\cos(0.5)$ using $P_4(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$

b) Since the terms are alternating, the error in the approximation is…

The actual value of $\cos(0.5)$ must be between:

Ex 4: Let *f* be a function that has derivatives of all orders.

Assume $f(3) = 1$, $f'(3) = \frac{1}{2}$, $f''(3) = \frac{-1}{4}$, $f'''(3) = \frac{3}{8}$ and the graph of $f^{(4)}(x)$ on $[3, 3.7]$ is shown.



a) Find $P_3(x)$, the 3rd degree Taylor polynomial, about $x = 3$ for the function *f*.

b) Use $P_3(x)$ to estimate $f(3.7)$.

c) Use the Lagrange error bound to show that $\left| f(3.7) - P_3(3.7) \right| < \frac{1}{10}$.

9-#9 HW: Use separate paper. All of you…if you want any credit at all.

1) Find the 4th degree Taylor Polynomial for $f(x) = \frac{1}{x}$ centered at $x = 1$.

2) Find the 5th degree Taylor Polynomial for $f(x) = \sin(x)$ centered at $x = \pi$.

3) Using just the 3rd degree polynomial from problem (2)
   a) Approximate $\sin(2.4)$
   b) Use the alternating error bound to find the error bound for $\sin(2.4)$.
   c) Use the error bound to find an interval $[a, b]$ such that $a \le \sin(2.4) \le b$.
   d) If Lagrange was used to find the error it would come out to be the same value. Why?

4) Find the 4th degree Taylor Polynomial for $f(x) = \sqrt{x}$ centered at $x = 1$.

5) Using just the 3rd degree polynomial from problem (4)
   a) Approximate $\sqrt{0.7}$
   b) Use the Lagrange error bound to find the error in $\sqrt{0.7}$.
   c) Use the error bound to find an interval $[a, b]$ such that $a \le \sqrt{0.7} \le b$.

6) Find the 3rd degree Taylor Polynomial for $f(x) = \frac{2}{x^2}$ centered at $x = 2$.