7 - #10   p. 483, 3, 6, 9, 15, 20, 21, 39, 43, 52   no extras

no calc

$\int \csc x \, dx = -\ln(\csc x + \cot x)$

$\text{arc} = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

3. $y = \frac{2}{3} x^{3/2} + 1$

$y' = x^{1/2}$

$\int_0^1 \sqrt{1 + (x^{1/2})^2}\, dx$

$\int_0^1 \sqrt{1+x}\, dx$ $u = 1 + x$, $du = dx$

$\int_1^2 \sqrt{u}\, du$

$\frac{2}{3}(u^{3/2})\Big|_1^2 = \boxed{\frac{2}{3} \cdot 2^{3/2} - \frac{2}{3}} = 1.219$

6. $y = \frac{x^4}{8} + \frac{1}{4x^2}$ $\frac{1}{4} x^{-2}$

$y' = \frac{4}{8} x^3 - \frac{1}{2} x^{-3} = \frac{x^3}{2} - \frac{1}{2x^3}$

$= \frac{x^6 - 1}{2x^3}$

$[y']^2 = \frac{x^{12} - 2x^6 + 1}{4x^6}$

$\int_1^2 \sqrt{1 + \frac{x^{12} - 2x^6 + 1}{4x^6}}$

$\int_1^2 \sqrt{\frac{4x^6}{4x^6} + \frac{x^{12} - 2x^6 + 1}{4x^6}}$

$\int_1^2 \sqrt{\frac{x^{12} + 2x^6 + 1}{4x^6}}$

$\int_1^2 \sqrt{\frac{(x^6+1)^2}{(2x^3)^2}}$

$\int_1^2 \frac{x^6 + 1}{2x^3}$

$\int_1^2 \frac{1}{2} x^3 + \frac{1}{2} x^{-3}\, dx$

$\left[\frac{1}{8} x^4 - \frac{1}{4} x^{-2}\right]_1^2$

$2 - \frac{1}{16} - \frac{1}{8} + \frac{1}{4}$

$\frac{32 - 1 - 2 + 4}{16}$

$\boxed{\frac{33}{16}}$

9. $y = \ln(\sin x)$ $[\pi/4, 3\pi/4]$

$y' = \frac{\cos x}{\sin x} = \cot(x)$

$\int_{\pi/4}^{3\pi/4} \sqrt{1 + \cot^2 x}\, dx$

$\int_{\pi/4}^{3\pi/4} \sqrt{\csc^2 x}\, dx$

$\int_{\pi/4}^{3\pi/4} \csc x\, dx$

$\left[-\ln|\csc x + \cot x|\right]_{\pi/4}^{3\pi/4}$

$-\ln|(\sqrt{2} - 1) + \ln|\sqrt{2} + 1||$

$\ln(\sqrt{2} + 1) - \ln(\sqrt{2} - 1)$

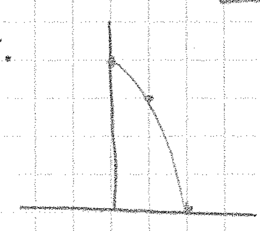
$\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ $\frac{\sqrt{2}+1}{\sqrt{2}+1}$

$\ln\left(\frac{(\sqrt{2}+1)^2}{2-1}\right)$

$\ln[(\sqrt{2}+1)^2]$

$\boxed{2\ln(\sqrt{2}+1)}$

15.

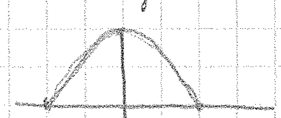


$y = 4 - x^2$

$y' = -2x$

$\int_0^2 \sqrt{1 + 4x^2}\, dx = 4.647$

20. $y = \cos x$ $-\pi/2 \le x \le \pi/2$



$y' = -\sin x$

$\int_{-\pi/2}^{\pi/2} \sqrt{1 + \sin^2 x}\, dx = 3.820$

21. $x = e^{-y}$ $0 \le y \le 2$

$\ln(x) = -y$ $1 \ge x \ge e^{-2}$

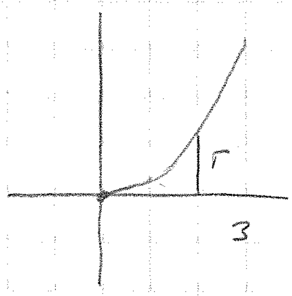
$y = -\ln(x)$ $e^{-2} \le x \le 1$

$y' = -1/x$

$\int_{e^{-2}}^{1} \sqrt{1 + 1/x^2}$

$= 2.221$

39. $y = \frac{1}{3}x^3$ $\quad x^2$



$$S = 2\pi\int_a^b r(x)\sqrt{1+[f'(x)]^2}\,dx$$

$$= 2\pi\int_0^3 \tfrac{1}{3}x^3\sqrt{1+x^4}\,dx \qquad u = 1+x^4,\quad du = 4x^3\,dx,\quad \tfrac{1}{4}du = x^3dx$$

$$\frac{2\pi}{3}\cdot\frac{1}{4}\int_1^{82} u^{1/2}\,du$$

$$\pi/6\left[\tfrac{2}{3}u^{3/2}\right]_1^{82}$$

$$\pi/6\left[\tfrac{2}{3}(82)^{3/2} - \tfrac{2}{3}\right]$$

$$\boxed{\pi/9\left[82\sqrt{82}-1\right]}$$

43. $y = \sqrt[3]{x} + 2$

$y - 2 = \sqrt[3]{x}$

$x = (y-2)^3 \qquad dx/dy = 3(y-2)^2$

$$2\pi\int_3^4 (y-2)^3\sqrt{1+9(y-2)^4}\,dy \qquad u = y-2,\quad du = dy$$

$$2\pi\int_1^2 u^3\sqrt{1+9u^4}\,du \qquad w = 1+9u^4,\quad dw = 36u^3\,du,\quad \tfrac{1}{36}dw = u^3du$$

$$+\frac{2\pi}{36}\int_{10}^{145}\sqrt{u}\,du$$

$$+\frac{\pi}{18}\left[\tfrac{2}{3}u^{3/2}\right]_{10}^{145}$$

$$+\frac{2\pi}{18\cdot 3}\left[145^{3/2} - 10^{3/2}\right]$$

$$\frac{\pi}{27}\left[145\sqrt{145} - 10\sqrt{10}\right]$$

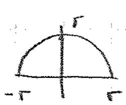
21. $x = e^{-y}$

$$\frac{dx}{dy} = e^{-y}\cdot -1$$

$$\int_0^2 \sqrt{1+e^{-2y}}\,dy$$

$$= 2.221$$

52. $y = \sqrt{r^2-x^2}$



$y' = \frac{1}{2}(r^2-x^2)^{-1/2}\cdot -2x$

$y' = \dfrac{-x}{\sqrt{r^2-x^2}}$

$[y']^2 = \dfrac{x^2}{r^2-x^2}$

$$2\pi\int_{-r}^{r}\sqrt{r^2-x^2}\sqrt{1+\frac{x^2}{r^2-x^2}}\,dx$$

$$2\pi\int_{-r}^{r}\sqrt{r^2-x^2}\cdot\sqrt{\frac{r^2\cancel{-x^2}\cancel{+x^2}}{r^2-x^2}}\,dx$$

$$2\pi\int_{-r}^{r}\cancel{\sqrt{r^2-x^2}}\cdot\frac{\sqrt{r^2}}{\cancel{\sqrt{r^2-x^2}}}\,dx$$

$$2\pi\int_{-r}^{r} r\,dx$$

$$2\pi\left[rx\right]_{-r}^{r}$$

$$2\pi\left[r^2-(-r^2)\right]$$

$$2\pi\left[2r^2\right]$$

$$\boxed{4\pi r^2}$$