Ch. 2 Test Review Solutions

1. $f(x) = e^{1/x}$

$f'(x) = e^{1/x} \cdot \frac{d}{dx}(x^{-1})$

$= e^{1/x} \cdot -x^{-2}$

$= \frac{-e^{1/x}}{x^2}$ **A**

2. $y = \cos^2(3x)$

$\frac{dy}{dx} = 2[\cos(3x)]^1 \cdot -\sin(3x) \cdot 3$

$= -6\cos(3x)\sin(3x)$ **A**

3. $f(x) = \ln(\ln(x))$

$f'(x) = \frac{1}{\ln(x)} \cdot \frac{1}{x}$ **E**

$f'(x) = \frac{1}{x\ln(x)}$

$f''(x) = \frac{x\ln(x)\cdot 0 - [1\cdot(x\cdot 1/x + \ln(x)]}{(x\ln(x))^2}$

$= \frac{0 - [1 + \ln(x)]}{(x\ln(x))^2}$

$= \boxed{\frac{-1 - \ln(x)}{[x\ln(x)]^2}}$

4. $y = \tan^{-1}(\cos(x))$ **A**

$y' = \frac{1}{1+\cos^2(x)} \cdot -\sin(x)$

5. $\frac{d}{dx}[2^x] = 2^x \cdot \ln(2)$ **C**

6. $\frac{d}{dx}[\ln|\cos(\pi/x)|]$

$\frac{1}{\cos(\pi/x)} \cdot -\sin(\pi/x) \cdot -\pi x^{-2}$

$\frac{\pi\tan(\pi/x)}{x^2}$ **E**

7. $x^2 + xy + y^3 = 0$

$2x + x\frac{dy}{dx} + 1y + 3y^2 \cdot \frac{dy}{dx} = 0$

$x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -2x - y$

$\frac{dy}{dx}(x + 3y^2) = -2x - y$

$\frac{dy}{dx} = -\frac{(2x+y)}{x+3y^2}$ **A**

8. $f(x) = \frac{x-1}{x+1}$

$f'(x) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$

$= \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2}$ **D**

$f'(1) = 2/2^2 = 2/4 = 1/2$

9. $y = \ln(x^2+y^2)$ $dy/dx|_{(1,0)}$

$y' = \frac{1}{x^2+y^2}(2x + 2yy')$

$y' = \frac{2x}{x^2+y^2} + \frac{2yy'}{x^2+y^2}$

$y' - \frac{2yy'}{x^2+y^2} = \frac{2x}{x^2+y^2}$

$y'\left(1 - \frac{2y}{x^2+y^2}\right) = \frac{2x}{x^2+y^2}$

$y' = \frac{2x}{x^2+y^2} \div \left(1 - \frac{2y}{x^2+y^2}\right)$

$y'|_{(1,0)} = \frac{2}{1} \div (1 - 0)$

$= 2 \div 1 = 2$ **D**

10. $f(x) = e^x$

$f'(x) = e^x$

$f'(2) = e^2$ **A**

$\ln|f'(2)| = \ln(e^2) = 2\ln e$

$= 2$

11. $h(x) = f(g(x))$
$h'(x) = f'(g(x)) \cdot g'(x)$
$h'(1) = f'(g(1)) \, g'(1)$
$= f'(2)(-3)$
$(-4)(-3)$
12 **D**

12. $f(x) = \sec(x)$
$\lim_{h\to 0} \frac{\sec(\pi/6+h) - \sec(\pi/6)}{h} = f'(\pi/6)$
$f'(x) = \sec(x)\tan(x)$
$f'(\pi/6) = (2/\sqrt{3})(1/\sqrt{3}) = 2/3$ **B**

13. $\lim_{x\to a} \frac{f(x) - f(2)}{x-2} = f'(2) = 0$ **B**
alternate form of def of deriv

14. $\sin(0) = (0)^2$ ✓ cont
$(1)^2 = 2-1$ ✓ cont
$2-2 \neq 2-3$ not cont at $x=2$
C

15. $\frac{dr}{dt} = .3$ in/sec
Find $\frac{dV}{dt}$ when $S = 100\pi$
$4\pi r^2 = 100\pi$
$r^2 = 25$
$r = 5$
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$
$= 4\pi(5)^2(3/10)$
$= \frac{100\pi \cdot 3}{10}$
$= 30\pi$ **E**

16. Find dx/dt when $dy/dt = 4/9$
$\frac{y}{2} = \frac{x+y}{8}$
$2x + 2y = 8y$
$2x = 6y$
$x = 3y$
$\frac{dx}{dt} = 3\frac{dy}{dt}$
$\frac{dx}{dt} = 3(4/9)$
$= 12/9 = 1/3$ **D**

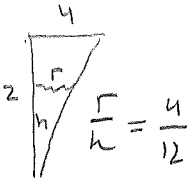
17. $\cos(x/y) = -2x$
$-\sin(x/y) \cdot \left[\frac{y \cdot 1 - x \cdot y'}{y^2}\right] = -2$
$\frac{y - xy'}{y^2} = 2 \cdot \frac{1}{\sin(x/y)}$
$y - xy' = 2y^2 \cdot \csc(x/y)$
$-xy' = 2y^2\csc(x/y) - y$
$y' = \frac{2y^2\csc(x/y) - y}{-x}$
$y' = \frac{y - 2y^2\csc(x/y)}{x}$ **A**

18. $\frac{dy}{dx} = y^{1/3}$
$\frac{d^2y}{dx^2} = \frac{1}{3}y^{-2/3} \cdot \frac{dy}{dx}$
$= \frac{1}{3}y^{-2/3} \cdot y^{1/3}$
$= \frac{1}{3}y^{-1/3} = \boxed{\frac{1}{3y^{1/3}}}$ **C**

19. $P(x) = x^{1/3}$ ← cont at $x=0$
$f'(x) = 1/3\,x^{-2/3}$ ← und at $x=0$
$= \frac{1}{3x^{2/3}}$

20. $\frac{dh}{dt} = h - 12$

a) $V = \frac{1}{3}\pi r^2 h$ $3r = h$, $r = 1/3\,h$
$V = 1/3\pi(1/3h)^2 h$
$\boxed{V = \pi/27\, h^3}$



$\frac{r}{h} = \frac{4}{12}$
$\frac{r}{h} = 1/3$

b) Find $\frac{dV}{dt}$ when $h=3$
$\frac{dV}{dt} = \pi/9\, h^2 \frac{dh}{dt}$
$\frac{dV}{dt} = \pi/9(3)^2(3-12) = \boxed{-9\pi \text{ ft}^3/\text{min}}$

c) $V = \pi r^2 y$ Find dy/dt when $h=3$
$V = 400\pi y$
$\frac{dV}{dt} = 400\pi \frac{dy}{dt}$
$9\pi = 400\pi \frac{dy}{dt}$
$\boxed{\frac{9}{400} \text{ ft/min}}$