

Ch. 2 Test Review Solutions

$$\begin{aligned}
 1. f(x) &= e^{1/x} \\
 f'(x) &= e^{1/x} \cdot \frac{d}{dx}(x^{-1}) \\
 &= e^{1/x} \cdot -x^{-2} \\
 &= -\frac{e^{1/x}}{x^2} \quad \boxed{A}
 \end{aligned}$$

$$\begin{aligned}
 2. y &= \cos^2(3x) \\
 \frac{dy}{dx} &= 2[\cos(3x)]^1 \cdot -\sin(3x) \cdot 3 \\
 &= -6\cos(3x)\sin(3x) \quad \boxed{A}
 \end{aligned}$$

$$\begin{aligned}
 3. f(x) &= \ln(\ln(x)) \\
 f'(x) &= \frac{1}{\ln(x)} \cdot \frac{1}{x} \\
 f'(x) &= \frac{1}{x \ln(x)} \quad \boxed{E}
 \end{aligned}$$

$$\begin{aligned}
 f''(x) &= \frac{x \ln(x) \cdot 0 - [1 \cdot (x \cdot \frac{1}{x} + \ln(x))]}{(x \ln(x))^2} \\
 &= \frac{0 - [1 + \ln(x)]}{(x \ln(x))^2} \\
 &= \frac{-1 - \ln(x)}{(x \ln(x))^2}
 \end{aligned}$$

$$\begin{aligned}
 4. y &= \tan^{-1}(\cos(x)) \quad \boxed{A} \\
 y' &= \frac{1}{1 + \cos^2(x)} \cdot -\sin(x)
 \end{aligned}$$

$$\begin{aligned}
 5. \frac{d}{dx} [2^x] &= 2^x \cdot \ln(2) \\
 &\quad \boxed{C}
 \end{aligned}$$

$$\begin{aligned}
 6. \frac{d}{dx} \left[\ln \left| \cos \left(\frac{\pi}{x} \right) \right| \right] \\
 \frac{1}{\cos(\pi/x)} \cdot -\sin(\pi/x) \cdot -\pi x^{-2} \\
 \frac{\pi \tan(\pi/x)}{x^2} \quad \boxed{E}
 \end{aligned}$$

$$\begin{aligned}
 7. x^2 + xy + y^3 &= 0 \\
 2x + x \frac{dy}{dx} + y + 3y^2 \cdot \frac{dy}{dx} &= 0 \\
 x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} &= -2x - y
 \end{aligned}$$

$$\begin{aligned}
 \frac{dy}{dx} (x + 3y^2) &= -2x - y \\
 \frac{dy}{dx} &= -\frac{(2x + y)}{x + 3y^2} \quad \boxed{A}
 \end{aligned}$$

$$\begin{aligned}
 8. f(x) &= \frac{x-1}{x+1} \\
 f'(x) &= \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2} \\
 &= \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2} \quad \boxed{D}
 \end{aligned}$$

$$f'(1) = \frac{2}{2^2} = \frac{2}{4} = \frac{1}{2}$$

$$\begin{aligned}
 9. y &= \ln(x^2 + y^2) \quad \frac{dy}{dx} \Big|_{(1,0)} \\
 y' &= \frac{1}{x^2 + y^2} (2x + 2y y')
 \end{aligned}$$

$$y' = \frac{2x}{x^2 + y^2} + \frac{2y y'}{x^2 + y^2}$$

$$y' - \frac{2y y'}{x^2 + y^2} = \frac{2x}{x^2 + y^2}$$

$$y' \left(1 - \frac{2y}{x^2 + y^2} \right) = \frac{2x}{x^2 + y^2}$$

$$y' = \frac{2x}{x^2 + y^2} \div \left(1 - \frac{2y}{x^2 + y^2} \right)$$

$$\begin{aligned}
 y' \Big|_{(1,0)} &= \frac{2}{1} \div (1 - 0) \\
 &= 2 \div 1 = 2 \quad \boxed{D}
 \end{aligned}$$

$$\begin{aligned}
 10. f(x) &= e^x \\
 f'(x) &= e^x \\
 f'(2) &= e^2 \\
 \ln(f'(2)) &= \ln(e^2) = 2 \ln e \\
 &= 2 \quad \boxed{A}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad w(x) &= f(g(x)) \\
 w'(x) &= f'(g(x)) \cdot g'(x) \\
 w'(1) &= f'(g(1)) \cdot g'(1) \\
 &= f'(2) \cdot (-3) \\
 &= (-4) \cdot (-3) \\
 &= 12 \quad \boxed{D}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad f(x) &= \sec(x) \\
 \lim_{h \rightarrow 0} \frac{\sec(\pi/6+h) - \sec(\pi/6)}{h} &= f'(\pi/6) \\
 f'(x) &= \sec(x) \tan(x) \\
 f'(\pi/6) &= (2/\sqrt{3})(1/\sqrt{3}) = 2/3 \quad \boxed{B}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} &= f'(2) = 0 \quad \boxed{B} \\
 &\text{alternate form of def of deriv}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \sin(0) &= (0)^2 \quad \checkmark \text{ cont} \\
 (1)^2 &= 2-1 \quad \checkmark \text{ cont} \\
 2-2 &\neq 2-3 \quad \text{not cont} \\
 &\text{at } x=2
 \end{aligned}$$

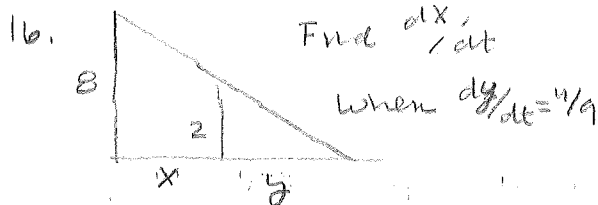
\boxed{C}

$$15. \quad \frac{dr}{dt} = .3 \text{ in/sec}$$

Find $\frac{dV}{dt}$ when $V = 100\pi$

$$\begin{cases}
 4\pi r^2 = 100\pi \\
 r^2 = 25 \\
 r = 5
 \end{cases}$$

$$\begin{aligned}
 \frac{dV}{dt} &= 4\pi r^2 \frac{dr}{dt} \\
 &= 4\pi(5)^2(3/10) \\
 &= \frac{10}{10} \pi \cdot 3 \\
 &= 30\pi \quad \boxed{E}
 \end{aligned}$$



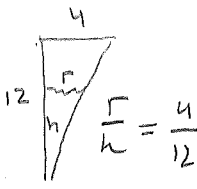
$$\begin{aligned}
 \frac{y}{2} &= \frac{x+y}{8} \\
 2x+2y &= 8y \\
 2x &= 6y \\
 x &= 3y \\
 \frac{dx}{dt} &= 3 \frac{dy}{dt} \\
 &= 3 \left(\frac{4}{9} \right) \\
 &= \frac{12}{9} = \frac{4}{3} \quad \boxed{D}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad \cos(x/y) &= -2x \\
 -\sin(x/y) \cdot \left[\frac{y \cdot 1 - x \cdot y'}{y^2} \right] &= -2 \\
 \frac{y - x y'}{y^2} &= 2 \cdot \frac{1}{\sin(x/y)}
 \end{aligned}$$

$$\begin{aligned}
 y - x y' &= 2y^2 \cdot \csc(x/y) \\
 -x y' &= 2y^2 \csc(x/y) - y \\
 y' &= \frac{2y^2 \csc(x/y) - y}{-x} \\
 y' &= \frac{y - 2y^2 \csc(x/y)}{x} \quad \boxed{A}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad \frac{dy}{dx} &= y^{1/3} \\
 \frac{d^2y}{dx^2} &= \frac{1}{3} y^{-2/3} \cdot \frac{dy}{dx} \\
 &= \frac{1}{3} y^{-2/3} \cdot y^{1/3} \\
 &= \frac{1}{3} y^{-1/3} = \frac{1}{3y^{1/3}} \quad \boxed{C}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad P(x) &= x^{1/3} \quad \leftarrow \text{cont at } x=0 \\
 P'(x) &= \frac{1}{3} x^{-2/3} \quad \leftarrow \text{und at } x=0 \\
 P'(0) &= \frac{1}{3x^{2/3}}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad \frac{dh}{dt} &= h - 12 \\
 a) \quad V &= \frac{1}{3} \pi r^2 h \quad \begin{cases} 3r = h \\ r = 1/3 h \end{cases} \\
 V &= \frac{1}{3} \pi (1/3 h)^2 h \\
 V &= \frac{\pi}{27} h^3 \quad \frac{r}{h} = 1/3
 \end{aligned}$$


b) Find $\frac{dV}{dt}$ when $h=3$

$$\frac{dV}{dt} = \pi/9 h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = \pi/9 (3)^2 (3-12) = -9\pi \text{ ft}^3/\text{min} \quad \boxed{}$$

c) $V = \pi r^2 h$ Find $\frac{dh}{dt}$ when $h=3$

$$\begin{aligned}
 V &= 400\pi \\
 \frac{dV}{dt} &= 400\pi \frac{dh}{dt} \\
 9\pi &= 400\pi \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{9}{400} \text{ ft/min} \quad \boxed{}
 \end{aligned}$$