Ch. 9 Test Review Multiple Choice

1. $f(x) = \sin(x^2)$

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

$\sin(x^2) = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!}$

$= x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!}$ $\boxed{A}$

2. $e^x = 1 + x + x^2/2 + x^3/3!$

$e^{(3x)} = 1 + (3x) + \frac{(3x)^2}{2} + \frac{(3x)^3}{3!}$

$= 1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{6}$

$\frac{27}{6} = \frac{9}{2}$ $\boxed{E}$

3. $\sin(t) = t - t^3/3! + t^5/5!$

$\frac{\sin(t)}{t} = 1 - t^2/3! + t^4/5!$ $\boxed{A}$

4. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n}$ $a = 9/5$ $r = -3/5$

$\frac{9}{5} + \frac{-27}{25} + \frac{81}{125}$

$\frac{a}{1-r} = \frac{9/5}{1 + 3/5}$ $\boxed{D}$

$9/5 \cdot 5/8 = \boxed{9/8}$

5. $\frac{f'''(4)(x-4)^3}{3!} = \frac{(x-4)^3}{512}$

$\frac{f'''(4)}{3!} = \frac{1}{512}$

$f'''(4) = \frac{6}{512}$

$= \frac{3}{256}$ $\boxed{D}$

6. $\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x+3/2)^{n+1}}{n+1} \cdot \frac{n}{(-1)^n (x+3/2)^n} \right|$

$\lim_{n\to\infty} \left| \frac{(x+3/2)\, n}{n+1} \right| = |x + 3/2|$

$|x + 3/2| < 1$

$-1 < x + 3/2 < 1$

$-5/2 < x < -1/2$

$x = -5/2$ $\sum \frac{(-1)^n(-1)^n}{n} = \sum 1/n$ diverge

$x = -1/2$ $\sum \frac{(-1)^n (1)^n}{n}$ = conv. alt series

$-5/2 < x \le -1/2$ $\boxed{B}$

7. $f(x) = (1-x)^{-2}$

$f'(x) = -2(1-x)^{-3} \cdot -1 = 2(1-x)^{-3}$

$f''(x) = -6(1-x)^{-4} \cdot -1 = +6(1-x)^{-4}$

$f'''(x) = -24(1-x)^{-5} \cdot -1 = +24(1-x)^{-4}$

$f(0) = 1$ $\frac{f(0)x^0}{0!} + \frac{f'(0)x^1}{1!}$

$f'(0) = 2$

$f''(0) = 6$

$f'''(0) = 24$

$1 + 2x + \frac{6x^2}{2!} + \frac{24x^3}{3!}$

$1 + 2x + 3x^2 + 4x^3$ $\boxed{C}$

8. $\frac{e^{3x^2}}{2}$ $x = 0$

$e^x = 1 + x + x^2/2 + x^3/3! + \dots$

$e^{(3x^2)} = 1 + (3x^2) + \frac{(3x^2)^2}{2} + \frac{(3x^2)^3}{3!}$

$= 1 + \dots + \frac{27x^6}{6}$

$\frac{e^{(3x^2)}}{2} = 1/2 + \dots \frac{27x^6}{6 \cdot 2}$

$\frac{27}{12} = \boxed{9/4}$ $\boxed{C}$

9. I. $\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2} < \sum_{n=1}^{\infty} \frac{1}{n^2}$

converges

II $\sum_{n=1}^{\infty} e^{-n} = \sum_{n=1}^{\infty} (1/e)^n$

geo $|r| = 1/e < 1$

converges

III $\sum_{n=1}^{\infty} \frac{n+2}{n^2+n}$

limit compare $\sum 1/n$

$\lim_{n\to\infty} \frac{n+2}{n^2+n} \cdot \frac{n}{1} = 1$

both diverge

I and II only $\boxed{D}$

10. If $\sum a_n$ converges

then if $b_n \le a_n$

b_n converges $\boxed{C}$

11. $a_n = \frac{(-1)^n}{\sqrt{n} + (-1)^n}$

$\sum_{n=2}^{\infty} a_n = \frac{1}{\sqrt{2}+1} - \frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{4}+1}$

$\frac{1}{1.4+1}$ $\frac{1}{1.7-1}$

I. alternating true

II. false $\frac{1}{2.41} < \frac{1}{.7}$

III. $\lim_{n\to\infty} a_n = 0$ true $\boxed{D}$

12. The sum of the series is the $\lim_{n\to\infty} S_n$

$\lim_{n\to\infty} (-1)^{n+1} = \text{DNE}$ $\boxed{E}$

the series diverges

13. $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$ just write out first terms

$\frac{-2^1}{e^2} + \frac{(-2)^2}{e^3} + \frac{(-2)^3}{e^4}$

$\frac{-2}{e^2} + \frac{4}{e^3} - \frac{8}{e^4}$ $a = -2/e^2$ $r = -2/e$

$\frac{\frac{-2}{e^2}}{1 + \frac{2}{e}} = \frac{\frac{-2}{e^2}}{\frac{e+2}{e}} = \frac{-2}{e^2} \cdot \frac{e}{e+2} = \boxed{\frac{-2}{e(e+2)}}$ $\boxed{B}$

14. $\frac{f^{(4)}(0) \cdot x^4}{4!} = 6x^4$

$\frac{f^{(4)}(0)}{4!} = 6$

$f^{(4)}(0) = 6 \cdot 4!$
$= 6 \cdot 24$
$= 144$ $\boxed{E}$

15. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

$e^{(3x)} = 1 + (3x) + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!}$

$= 1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{6}$ $\boxed{E}$

16. I. no

II. no $\lim_{n\to\infty} \frac{1}{2n-1} = 1/2 \neq 0$

III. ~~geo~~ geo $|r| = 1/3$ $\boxed{C}$

17. $f(x) = \sin^2 x$
$f'(x) = 2\sin x \cdot \cos x = \sin(2x)$
$f''(x) = 2\cos(2x)$

$f(0) = 0$
$f'(0) = 0$
$f''(0) = 2$

$\frac{f''(0)x^2}{2!}$

$\frac{2x^2}{2} = 1x^2$

$\boxed{D}$

18. $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = e^x$

$1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \dots = e^2$ $\boxed{B}$

19. $f''(x) = \sqrt{1+3x}$
$f'''(x) = 1/2(1+3x)^{-1/2} \cdot 3$
$= \frac{3}{2\sqrt{1+3x}}$

$\frac{f'''(0)x^3}{3!} = \frac{3/2\, x^3}{6} = \frac{1}{4}x^3$ $\boxed{C}$

20. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-4)^n}{2\cdot 3^{n+1}}$

$\displaystyle\lim_{n\to\infty}\left|\frac{(x-4)^{n+1}}{2\cdot 3^{n+2}}\cdot\frac{3^{n+1}\cdot 2}{(x-4)^n}\right|$

$\displaystyle\lim_{n\to\infty}\left|\frac{(x-4)\ 3^n\cdot 3}{3^n\cdot 3^2}\right| = \frac{1}{3}|x-4|$

$\frac{1}{3}|x-4|<1$

$|x-4|<3$ radius $=3$

$\boxed{C}$

21. error $\displaystyle\le \frac{\max|f^{(4)}(z)|\cdot x^4}{4!}$

$\displaystyle\le \frac{\frac{4}{4+1}\cdot(1)^4}{4!}$

$\displaystyle\le \frac{4}{5}\cdot\frac{1}{4!}\cdot 1$ $\boxed{B}$

22. $g(x)=\int_0^x f(t)\,dt$

$f(x)= 3-4x+\frac{2x^2}{2!}+\frac{x^3}{3!}$

$g(x) = 3x-2x^2+\frac{2}{2!}\cdot\frac{1}{3}x^3+\frac{1}{3!}\cdot\frac{1}{4}x^4$

$= 3x-2x^2+\frac{1}{3}x^3+\dots$

$\boxed{C}$

23. The sum of the series is the limit of the partial sum.

$\displaystyle\lim_{n\to\infty}\frac{n}{3n+1}=\frac{1}{3}$

$\displaystyle\sum_{k=1}^{\infty} a_k = \frac{1}{3}$ $\boxed{A}$

24. I. no $|a_n|$ is not strictly decreasing

II. yes

III no $\lim_{n\to\infty}|a_n| = \frac{1}{2}\neq 0$

$\boxed{B}$

FRQ: 2007 Q6:

a) $1+x+x^2/2!+x^3/3!+\dots = e^x$

$e^{-x^2} = 1+(-x^2)+\frac{(-x^2)^2}{2!}+\frac{(-x^2)^3}{3!}+\dots$

$= 1-x^2+\frac{x^4}{2!}-\frac{x^6}{3!}+\dots\frac{(-1)^n x^{2n}}{n!}$

b) $\displaystyle\lim_{x\to 0}\frac{1-x^2-(1-x^2+x^4/2!-x^6/3!+\dots)}{x^4}$

$\displaystyle\lim_{x\to 0} -\frac{1}{2}+\frac{x^2}{3!}+\dots = \boxed{-\frac{1}{2}}$

c) $\displaystyle\int_0^x e^{-t^2}dt = \left[t-\frac{1}{3}t^3+\frac{1}{10}t^5-\frac{1}{42}t^7\right]_0^x$

$x-\frac{1}{3}x^3+\frac{1}{10}x^5-\frac{1}{42}x^7$

$\displaystyle\int_0^{1/2} e^{-t^2}dt \approx \frac{1}{2}-\frac{1}{3}\left(\frac{1}{2}\right)^3$

$= \frac{1}{2}-\frac{1}{3}\cdot\frac{1}{8} = \frac{1}{2}-\frac{1}{24}$

d) The error is less then the first omitted term.

error $< \frac{1}{10}\left(\frac{1}{2}\right)^5 = \frac{1}{320} < \frac{1}{200}$

I used the error bound for alternating series since the series is alternating and the |terms| are decreasing.

FRQ 2: → next page for scoring guide

watch video of set scoring guide

search "2009 Q6 BC Calc"

Question 6

The Maclaurin series for e^x is $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \cdots$. The continuous function f is defined by $f(x) = \frac{e^{(x-1)^2} - 1}{(x-1)^2}$ for $x \neq 1$ and $f(1) = 1$. The function f has derivatives of all orders at $x = 1$.

(a) Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about $x = 1$.

(b) Use the Taylor series found in part (a) to write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.

(c) Use the ratio test to find the interval of convergence for the Taylor series found in part (b).

(d) Use the Taylor series for f about $x = 1$ to determine whether the graph of f has any points of inflection.

(a) $1 + (x-1)^2 + \frac{(x-1)^4}{2} + \frac{(x-1)^6}{6} + \cdots + \frac{(x-1)^{2n}}{n!} + \cdots$	$2 : \begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$
(b) $1 + \frac{(x-1)^2}{2} + \frac{(x-1)^4}{6} + \frac{(x-1)^6}{24} + \cdots + \frac{(x-1)^{2n}}{(n+1)!} + \cdots$	$2 : \begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$
(c) $\lim_{n\to\infty} \left\| \frac{\frac{(x-1)^{2n+2}}{(n+2)!}}{\frac{(x-1)^{2n}}{(n+1)!}} \right\| = \lim_{n\to\infty} \frac{(n+1)!}{(n+2)!}(x-1)^2 = \lim_{n\to\infty} \frac{(x-1)^2}{n+2} = 0$ Therefore, the interval of convergence is $(-\infty, \infty)$.	$3 : \begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{answer} \end{cases}$
(d) $f''(x) = 1 + \frac{4 \cdot 3}{6}(x-1)^2 + \frac{6 \cdot 5}{24}(x-1)^4 + \cdots + \frac{2n(2n-1)}{(n+1)!}(x-1)^{2n-2} + \cdots$ Since every term of this series is nonnegative, $f''(x) \geq 0$ for all x. Therefore, the graph of f has no points of inflection.	$2 : \begin{cases} 1 : f''(x) \\ 1 : \text{answer} \end{cases}$