Ch. 2 Test Review Solutions

1. $f(x) = e^{1/x}$

$f'(x) = e^{1/x} \cdot \frac{d}{dx}(x^{-1})$

$= e^{1/x} \cdot -x^{-2}$

$= \frac{-e^{1/x}}{x^2}$ **A**

2. $y = \cos^2(3x)$

$\frac{dy}{dx} = 2[\cos(3x)]^1 \cdot -\sin(3x) \cdot 3$

$= -6\cos(3x)\sin(3x)$ **A**

3. $f(x) = \ln(\ln(x))$

$f'(x) = \frac{1}{\ln(x)} \cdot \frac{1}{x}$ **E**

$f'(x) = \frac{1}{x\ln(x)}$

$f''(x) = \frac{x\ln(x)\cdot 0 - [1\cdot(x\cdot 1/x + \ln(x)]}{(x\ln(x))^2}$

$= \frac{0 - [1 + \ln(x)]}{(x\ln(x))^2}$

$= \boxed{\frac{-1 - \ln(x)}{[x\ln(x)]^2}}$

4. $y = \tan^{-1}(\cos(x))$ **A**

$y' = \frac{1}{1+\cos^2(x)} \cdot -\sin(x)$

5. $\frac{d}{dx}[2^x] = 2^x \cdot \ln(2)$

C

6. $\frac{d}{dx}[\ln|\cos(\pi/x)|]$

$\frac{1}{\cos(\pi/x)} \cdot -\sin(\pi/x) \cdot -\pi x^{-2}$

$\frac{\pi\tan(\pi/x)}{x^2}$ **E**

7. $x^2 + xy + y^3 = 0$

$2x + x\frac{dy}{dx} + 1y + 3y^2 \cdot \frac{dy}{dx} = 0$

$x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -2x - y$

$\frac{dy}{dx}(x + 3y^2) = -2x - y$

$dy/dx = -\frac{(2x+y)}{x+3y^2}$ **A**

8. $f(x) = \frac{x-1}{x+1}$

$f'(x) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$

$= \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2}$ **D**

$f'(1) = 2/2^2 = 2/4 = 1/2$

9. $y = \ln(x^2+y^2)$ $dy/dx|_{(1,0)}$

$y' = \frac{1}{x^2+y^2}(2x + 2yy')$

$y' = \frac{2x}{x^2+y^2} + \frac{2yy'}{x^2+y^2}$

$y' - \frac{2yy'}{x^2+y^2} = \frac{2x}{x^2+y^2}$

$y'\left(1 - \frac{2y}{x^2+y^2}\right) = \frac{2x}{x^2+y^2}$

$y' = \frac{2x}{x^2+y^2} \div \left(1 - \frac{2y}{x^2+y^2}\right)$

$y'|_{(1,0)} = \frac{2}{1} \div (1 - 0)$

$= 2 \div 1 = 2$ **D**

10. $f(x) = e^x$

$f'(x) = e^x$

$f'(2) = e^2$ **A**

$\ln|f'(2)| = \ln(e^2) = 2\ln e$

$= 2$

11. $h(x) = f(g(x))$
$h'(x) = f'(g(x)) \cdot g'(x)$
$h'(1) = f'(g(1)) g'(1)$
$= f'(2)(-3)$
$(-4)(-3)$
12 **D**

12. similar to
$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

D

13. $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$

C

14. $\sin(0) = (0)^2$ ✓ cont
$(1)^2 = 2 - 1$ ✓ cont
$2 - 2 \neq 2 - 3$ not cont at $x = 2$

C

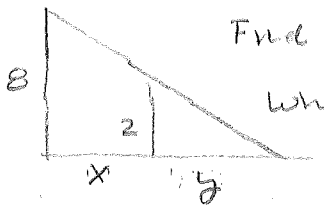
15. $\frac{dr}{dt} = .3$ in/sec

Find $\frac{dV}{dt}$ when $S = 100\pi$

$4\pi r^2 = 100\pi$
$r^2 = 25$
$r = 5$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$
$= 4\pi(5)^2(3/10)$
$= \frac{100\pi \cdot 3}{10}$
$= 30\pi$ **E**

16.



Find dx/dt when $dy/dt = 4/9$

$\frac{y}{2} = \frac{x+y}{8}$
$2x + 2y = 8y$
$2x = 6y$
$x = 3y$

$\frac{dx}{dt} = 3\frac{dy}{dt}$
$\frac{dx}{dt} = 3(4/9)$
$= 12/9 = 4/3$ **D**

17. $\cos(x/y) = -2x$
$-\sin(x/y) \cdot \left[\frac{y \cdot 1 - x \cdot y'}{y^2}\right] = -2$
$\frac{y - xy'}{y^2} = 2 \cdot \frac{1}{\sin(x/y)}$
$y - xy' = 2y^2 \cdot \csc(x/y)$
$-xy' = 2y^2 \csc(x/y) - y$
$y' = \frac{2y^2 \csc(x/y) - y}{-x}$
$y' = \frac{y - 2y^2 \csc(x/y)}{x}$ **A**

18. $\frac{dy}{dx} = y^{1/3}$
$\frac{d^2y}{dx^2} = \frac{1}{3} y^{-2/3} \cdot \frac{dy}{dx}$
$= \frac{1}{3} y^{-2/3} \cdot y^{1/3}$
$= \frac{1}{3} y^{-1/3} = \frac{1}{3y^{1/3}}$ **C**

19. $P(x) = x^{1/3}$ ← cont at $x = 0$
$P'(x) = 1/3 x^{-2/3}$ ← und at $x = 0$

20. $\frac{dh}{dt} = h - 12$

a) $V = \frac{1}{3}\pi r^2 h$ $3r = h$, $r = 1/3 h$
$V = 1/3\pi(1/3 h)^2 h$
$V = \pi/27 h^3$

$\frac{r}{h} = \frac{4}{12}$
$\frac{r}{h} = 1/3$

b) Find $\frac{dV}{dt}$ when $h = 3$
$\frac{dV}{dt} = \pi/9 h^2 \frac{dh}{dt}$
$\frac{dV}{dt} = \pi/9 (3)^2 (3 - 12) = -9\pi$ ft^3/min

c) $V = \pi r^2 y$ Find dy/dt when $h = 3$
$V = 400\pi y$
$\frac{dV}{dt} = 400\pi \frac{dy}{dt}$
$9\pi = 400\pi \frac{dy}{dt}$
$\frac{9}{400}$ ft/min