

Determine if each integral is improper. If it is rewrite it with appropriate with limits. Do not evaluate.

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| 1. $\int_0^8 x^{-1/3} dx$ | 2. $\int_{-\infty}^{\infty} \frac{x}{1+x^2} dx$ | 3. $\int_0^3 \frac{1}{x-2} dx$ |
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Evaluate the following integrals. Use proper notation on every step. Some may converge and some diverge. Show appropriate work to support your answer.

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| 4. $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$ | 5. $\int_1^{\infty} \frac{1}{x^2} dx$ |
| 6. $\int_e^{\infty} \frac{1}{x \ln^3 x} dx$ | 7. $\int_0^1 \frac{1}{x^2} dx$ |
| 8. $\int_{-\infty}^3 \frac{1}{x^2+9} dx$ | 9. $\int_0^{\pi/2} \tan \theta d\theta$ |

Evaluate the following limits:

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| 10. $\lim_{x \rightarrow 0^+} (\cot x - \csc x)$ | 11. $\lim_{x \rightarrow 0} x \ln(x)$ |
| <p>12. Multiple Choice: $\lim_{x \rightarrow 0} (1+2x)^{\csc x} =$</p> <p>(A) 0 (B) 1 (C) 2 (D) e (E) e^2</p> | |

Sequences and Series:

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| 13. | <p>Which of the following sequences converge?</p> <p>I. $\left\{ \frac{5n}{2n-1} \right\}$</p> <p>II. $\left\{ \frac{e^n}{n} \right\}$</p> <p>III. $\left\{ \frac{e^n}{1+e^n} \right\}$</p> <p>(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III</p> |
| 14. | <p>Which of the following series converge?</p> <p>I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} 3 \left(\frac{6}{7} \right)^n$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$</p> <p>(A) None (B) II only (C) III only (D) I and II only (E) I and III only</p> |
| 15. | <p>The sum of the infinite geometric series $\frac{3}{2} + \frac{9}{16} + \frac{27}{128} + \frac{81}{1,024} + \dots$ is</p> <p>(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50</p> |

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| 16. | <p>If $\lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?</p> <p>(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges</p> <p>(B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges</p> <p>(C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges</p> <p>(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges</p> <p>(E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges</p> |
| 17. | <p>Which of the following series are convergent?</p> <p>I. $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} + \dots$</p> <p>II. $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots$</p> <p>III. $1 - \frac{1}{3} + \frac{1}{3^2} - \dots + \frac{(-1)^{n+1}}{3^{n-1}} + \dots$</p> <p>(A) I only (B) III only (C) I and III only (D) II and III only (E) I, II, and III</p> |

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| <p>18. Find the sum of the series.</p> <p>a) $e - \frac{e^2}{5} + \frac{e^3}{25} - \frac{e^4}{125} + \dots$</p> <p>b) $\sum_{n=0}^{\infty} 2^{-n}$</p> <p>c) $\sum_{n=0}^{\infty} \left[\frac{1}{n+1} - \frac{1}{n+2} \right]$</p> |
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| <p>19. Determine if the series converges or diverges. Justify.</p> <p>a) $\sum_{n=0}^{\infty} (-2)^n$</p> <p>b) $\sum_{k=0}^{\infty} \frac{k^2}{2k^2 + 1}$</p> <p>c) $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots$</p> <p>d) $\frac{1}{2} + \frac{2}{5} + \frac{3}{10} + \frac{4}{17} + \dots + \frac{n}{n^2 + 1} + \dots$</p> |
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