Advanced ALGEBRA	**Imaginary Cubics**	Name ____________
Problem of the Week	10 points Polynomials	Per.___ Date__________

III. Decades earlier Scipione del Ferro derived a formula for solving cubics. He told it to his apprentice Antonio Fior, who boasted to other mathematicians he knew how to solve it. Hearing this, Niccolo Fontana, aka "Tartaglia," derived a more advanced formula. He told it to Cardan, who vowed secrecy, then later published it giving Tartaglia credit. To solve a *depressed cubic* $x^3 + px = q$, $x = \sqrt[3]{q/2+\sqrt{(q/2)^2+(\cancel{q}\,p/3)^3}} - \sqrt[3]{-q/2+\sqrt{(q/2)^2+(p/3)^3}}$.

a. For $x^3 - 15x = 4$ find p and q. $p = -15 \quad q = 4$

b. $\frac{q}{2} = \frac{4}{2} = 2 \quad \frac{p}{3} = \frac{-15}{3} = -5$

c. $(q/2)^2 = 2^2 = 4 \qquad (p/3)^3 = (-5)^3 = -125$

d. $(q/2)^2 + (p/3)^3 = -121 \qquad \sqrt{(q/2)^2+(p/3)^3} = \sqrt{-121} = \sqrt{121(-1)} = 11\sqrt{-1}$

$4 + {}^{-}125$

e. Fill in the numbers in the arithmetic expression for x:

$x = \sqrt[3]{2+\sqrt{4-125}} - \sqrt[3]{-2+\sqrt{4-125}} = \sqrt[3]{2+\sqrt{-121}} - \sqrt[3]{-2+\sqrt{-121}}$

IV. When applied to the Bombelli cubic, the Cardan formula stated that x had square roots of negative numbers. Yet, x was also equal to four. It was this problem, and not that of solving $x^2 = -1$, that gave legitimacy to imaginary numbers. Later in the 17th Century, Leonhard Euler gave the notation $i = \sqrt{-1}$, so that $i^2 = -1$ and $i^3 = -i$. But what of Cardan's formula for solving Bombelli's cubic? Was it really equal to 4?

a. $(-2+i)(-2+i) = 4-2i-2i+i^2 = 4-4i-1 = 3-4i$

b. $(-2+i)^3 = (3-4i)(-2+i) = -6+8i+3i-4i^2 = -6+4+11i = -2+11i$

c. $(2+i)^2 = 2^2+2\cdot 2i+i^2 = 4+4i-1 = 3+4i$

d. $(2+i)^3 = (3+4i)(2+i) = 6+8i+3i+4i^2 = 6-4+11i = 2+11i$

e. Use parts a-d to find $\sqrt[3]{2+11i} - \sqrt[3]{-2+11i} = 2+i-(-2+i) =$

~~2+11i~~

$2 - {}^{-}2 + i - i =$

$2+2+0 = 4$ ✓

yes!