Advanced ALGEBRA | **Imaginary Cubics** | Name ____________
Problem of the Week | 10 points Polynomials | Per.___ Date____________

I. In his book *Ars Magna,* Giralamo Cardano of 16th Century Italy posed the problem of finding two numbers whose sum is 10 and whose product is 40.

a. $x+y=$ 10 b. $xy=$ 40 c. Solve part a for y: $\boxed{y=10-x}$

d. Substitute y into part b to get a quadratic equation.

$x(10-x)=40$

$10x-x^2=40$

$\boxed{x^2-10x+40=0}$

e. Use the quadratic formula to show that both solutions involve the square root of a negative number. $a=1$ $b=-10$ $c=40$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\dfrac{--10\pm\sqrt{100-4\cdot1\cdot40}}{2\cdot1}=\dfrac{+10\pm}{2}\dfrac{\sqrt{100-160}}{2}$

$=+5\pm\dfrac{\sqrt{-60}}{2}=+5\pm\dfrac{2\sqrt{-15}}{2}=\boxed{5\pm\sqrt{-15}}$

f. Add: $\left(5+\sqrt{-15}\right)+\left(5-\sqrt{-15}\right)=$ $5+5+\sqrt{-15}-\sqrt{-15}=5+5+0=\boxed{10}$

g. Multiply: $\left(5+\sqrt{-15}\right)\left(5-\sqrt{-15}\right)=$

$5^2+5\sqrt{-15}-5\sqrt{-15}-\sqrt{-15}^2=25-(-15)=20+15=\boxed{40}$

II. Later in sixteenth century Italy, Rafael Bombelli considered $x^3=15x+4$.

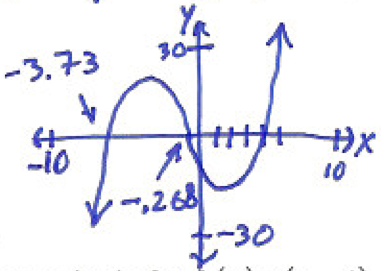
a. Write the equation as a polynomial $b(x)$ whose roots solve the equation.

$b(x)=x^3-15x-4=0$

b. Show that $x=4$ is a zero of the Bombelli cubic polynomial $b(x)$.

$b(4)=4^3-15(4)-4=64-60-4=0$

c. Graph the Bombelli cubic $y=b(x)$.



d. How many real roots does it have? 3

Three x-intercepts.

e. Use division (long or synthetic) to find a quadratic for $b(x)\div(x-4)$.

$$\begin{array}{r|rrrr} & 1 & 0 & -15 & -4 \\ 4 & & 4 & 16 & 4 \\ \hline & 1 & 4 & 1 & 0 \end{array}$$

$\boxed{x^2+4x+1}$

$$\begin{array}{r} x^2+4x+1 \\ x-4\overline{)x^3-15x-4} \\ x^3-4x^2 \\ \hline 4x^2-15x \\ 4x^2-16x \\ \hline x-4 \\ x-4 \\ \hline 0 \end{array}$$

f. Find the remaining roots in simple radical form using the quadratic formula.

$x=\dfrac{-4\pm\sqrt{16-4\cdot1}}{2}=\dfrac{-4\pm\sqrt{16-4}}{2}$

$=\dfrac{-4}{2}\pm\dfrac{\sqrt{12}}{2}=-2\pm\dfrac{2\sqrt{3}}{2}=\boxed{-2\pm\sqrt{3}}$

$-2-\sqrt{3}\doteq-3.73$

$-2+\sqrt{3}\doteq-.268$

$b(-3.73)\approx0$

$b(-.268)$

≈0