

6. The competition between the two banks is escalating and creating tedious work for everyone. They started compounding interest rates every hour, and then every minute, and finally, every second. Several factors complicated the process. The employees started to complain about carpal tunnel syndrome in their fingers from pressing all the calculator buttons. Many calculators broke and had to be replaced. Then interest rates changed and all the numbers previously worked out were no longer valid. Someone thought there must be a better way. So **Lenders' National** hired a mathematician to develop an equation that gives the principal balance P using the following variables:

d = initial deposit
 r = interest rate percent
 n = # of times compounded per year

- a) What is this equation?

$$P = d \left(1 + \frac{r}{n}\right)^n$$

7. Use this new equation to determine the balance for an account with:

- a) \$5,000 deposit on 8% compounded hourly = $5000 \left(1 + \frac{.08}{8760}\right)^{8760} = \$5,416.43$
 $24 \times 365 = 8760$ hours
- b) \$10,000 deposit on 5.5% compounded every minute = $10,000 \left(1 + \frac{.055}{525600}\right)^{525600} = \$10,565.41$
 $8760 \times 60 = 525,600$ minutes
- c) \$50,000 deposit on 3.25% compounded every second = $50,000 \left(1 + \frac{.0325}{31,536,000}\right)^{31,536,000} = \$51,651.75$
 $525600 \times 60 = 31,536,000$ seconds

8. Now the banks want to know if there is a limit if they compound 100% interest rate continuously (that is infinitely many times per year). The **Bank of e** wants you to find the limit "e" by filling in the table.

	A	B	C
1	COMPOUNDED	n	$(1 + 1/n)^n$
2	annually	1	2
3	semi-annually	2	$\left(1 + \frac{1}{2}\right)^2 = 2.25$
4	3 times/year	3	2.3703703704
5	quarterly	4	$\left(\frac{5}{4}\right)^4 = 2.44$
6	bi-monthly	6	2.5216
7		12	2.6130352902
8		52	2.69259695
9	daily	365	2.714567
10	hourly	8760	2.71812669
11	every minute	525600	2.7182792426
12	every second	31536000	2.718282493

9. a) What value is e?

$$e \approx 2.718281828...$$

- b) How is this number like π ?

irrational, non-repeating non-terminating decimal

10. Punch these buttons in your calculator to find \$1,000 compounded continuously at 12%.

- a) $1000 \times e^{0.12} =$

$$1127.496852$$

$$\approx \$1127.50$$

- b) How does this amount compare with all other methods of compounding?

Formula calculates most amount of interest \$1000 can earn at 12% compounded continuously.