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| Advanced ALGEBRA | **Imaginary Cubics** | Name ____________________ |
| Problem of the Week | 10 points Radicals | Per.___Date_______________ |

I. In his book *Ars Magna*, Giralamo Cardano of 16th Century Italy posed the problem of finding two numbers whose sum is 10 and whose product is 40.

a. $x + y =$ b. $xy =$ c. Solve part a for *y*:

d. Substitute *y* into part b to get a quadratic equation.

e. Use the quadratic formula to show that both solutions involve the square root of a negative number. Don't use *i* in your answer. Put in simple radical form.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

f. Combine like terms: $\left(5 + \sqrt{-15}\right) + \left(5 - \sqrt{-15}\right) =$

g. Expand (use difference of squares): $\left(5 + \sqrt{-15}\right) \times \left(5 - \sqrt{-15}\right) =$

II. Later in sixteenth century Italy, Rafael Bombelli considered $x^3 = 15x + 4$.

a. Write the equation as a polynomial $b(x)$ whose roots solve the equation.

b. Show that $x = 4$ is a zero of the Bombelli cubic polynomial $b(x)$.

c. Graph the Bombelli cubic $y = b(x)$.

d. How many real roots does it have?

e. Use division (long or synthetic) to find a quadratic for $b(x) \div (x - 4)$.

(Remember if the factor is $x - 4$, then the root is $^{+}4$.)

f. Find the remaining roots in simple radical form using the quadratic formula.

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III. Decades earlier Scipione del Ferro derived a formula for solving cubics. He told it to his apprentice Antonio Fior, who boasted to other mathematicians he knew how to solve it. Hearing this, Niccolo Fontana, aka "Tartaglia," derived a more advanced formula. He told it to Cardan, who vowed secrecy, then later published it giving Tartaglia credit. To solve a *depressed cubic* $x^3 + px = q$, $x = \sqrt[3]{q/2 + \sqrt{(q/2)^2 + (p/3)^3}} - \sqrt[3]{-q/2 + \sqrt{(q/2)^2 + (p/3)^3}}$.

a. For $x^3 - 15x = 4$ find *p* and *q*.

b. $\frac{q}{2} =$ $\frac{p}{3} =$

c. $(q/2)^2 =$ $(p/3)^3 =$

d. $(q/2)^2 + (p/3)^3 =$ $\sqrt{(q/2)^2 + (p/3)^3} = \sqrt{\phantom{xxxxx}}$

e. Fill in the numbers in the arithmetic expression for *x*:

$x = \sqrt[3]{\phantom{xx} + \sqrt{\phantom{xxxx}}} - \sqrt[3]{\phantom{xx} + \sqrt{\phantom{xxxx}}}$

IV. When applied to the Bombelli cubic, the Cardan formula stated that *x* had square roots of negative numbers. Yet, *x* was also equal to four. It was this problem, and not that of solving $x^2 = -1$, that gave legitimacy to imaginary numbers. Later in the 17th Century, Leonhard Euler gave the notation $i = \sqrt{-1}$, so that $i^2 = -1$ and $i^3 = -i$. But what of Cardan's formula for solving Bombelli's cubic? Was it really equal to 4?

a. $(-2+i)(-2+i)$

b. $(3-4i)(-2+i) =$

c. $(2+i)^2 =$

d. $(2+i)^3 =$

e. Use parts b & d to find $\sqrt[3]{2+11i} - \sqrt[3]{-2+11i} =$