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Web Resources
www.mathwarehouse.com/law-of-sines

ambiguous case - video Tutorial and practice problems
Law of Sines -video Tutorial and practice problems
Pictures of Law of Sines (problem diagrams and formula)
Law of Sines and Cosines Worksheets

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Law of Sines: Ambiguous Case

For any \( \Delta ABC \):

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

or

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

I. Model Problems

In the following example you will find the possible measures of an angle given the sine of the angle.

**Example 1:**

*Find the measure of \( \angle A \) if \( \sin A = \frac{1}{2} \).*

Shown in Quadrant I is angle \( A \) with a sine of \( \frac{1}{2} \).

Finding the inverse, \( \angle A = 30^\circ \).

In Quadrant II is another angle \( A \) with a sine of \( \frac{1}{2} \).

\( \angle A = 180^\circ - 30^\circ \)

\( \angle A = 150^\circ \)

Therefore if \( \sin A = \frac{1}{2} \), then \( \angle A = 30^\circ \) or \( 150^\circ \).
In the following example you will find all the possible measures of an angle of a triangle using Law of Sines.

Example 2:
For \( \triangle ABC, \ a = 8, b = 10, \ \text{and} \ m \angle A = 34^\circ \). Find all the possible \( m \angle B \) to the nearest whole degree.

Write down known. \( a = 8, b = 10, \ m \angle A = 34^\circ \)

Draw and label a figure. Drawn is one possible triangle (in this case an acute triangle). There may be more than one possible triangle.

Law of Sines \[ \frac{\sin B}{b} = \frac{\sin A}{a} \]

Substitute. \[ \frac{\sin B}{10} = \frac{\sin 34^\circ}{8} \]

Isolate \( \sin B \). \[ \sin B = \left(\frac{\sin 34^\circ}{8}\right)(10) \]

Find the inverse. \[ \sin B = \frac{10}{8} \sin 34^\circ \]

Round to the nearest whole degree. \( m \angle B \approx 44^\circ \)

Find \( m \angle B \) in Quadrant II \( m \angle B \approx 180^\circ - 44^\circ \)

Check to see if 136° is a possible angle measurement for the triangle using the sum of the angles of a triangle.

\[ m \angle A + m \angle B \approx 180^\circ? \]
\[ 44^\circ + 136^\circ \approx 180^\circ? \]
\[ 170^\circ \approx 180^\circ \checkmark \]
II. Practice Problems

1. For $\triangle ABC$, 
   $a = 7, b = 15$, and $\angle A = 26^\circ$. Find all possible $\angle B$ to the nearest degree.

2. For $\triangle ABC$, 
   $b = 17, c = 24$, and $\angle B = 16^\circ$. Find all possible $\angle C$ to the nearest degree.

3. For $\triangle DEF$, 
   $e = 52, f = 41$, and $\angle F = 48^\circ$. Find all possible $\angle E$ to the nearest degree.

4. For $\triangle LMN$, 
   $l = 27, m = 15$, and $\angle L = 55^\circ$. Find all possible $\angle M$ to the nearest degree.

5. For $\triangle ABC$, 
   $b = 120, c = 92$, and $\angle C = 42^\circ$. How many triangles can be formed?

6. For $\triangle DEF$, 
   $d = 6, e = 24$, and $\angle E = 38^\circ$. How many Triangles can be formed?
III. Practice (harder) & word problems

7. For $\triangle ABC$,
   $a = 62, b = 53, \text{ and } m\angle A = 54^\circ$. Find all possible $m\angle A$ to the nearest degree.

8. For $\triangle LMN$,
   $m = 8, n = 11, \text{ and } m\angle M = 6^\circ$. Find all possible $m\angle N$ to the nearest degree.

9. For $\triangle XYZ$, $y = 7, z = 5, \text{ and } m\angle Y = 19^\circ$. Find all possible $m\angle Z$ to the nearest degree.

10. For $\triangle ABC$,
    $a = 40, c = 49, \text{ and } m\angle C = 32^\circ$. Find all possible $m\angle A$ to the nearest degree.

11. For $\triangle DEF$,
    $e = 12, f = 21, \text{ and } m\angle E = 25^\circ$. Find all possible $m\angle F$ to the nearest degree.

12. For $\triangle LMN$,
    $l = 30, m = 24, \text{ and } m\angle M = 40^\circ$. Find all possible $m\angle L$ to the nearest degree.

13. A triangle has two sides with lengths of 20 and 15. The measure of the angle opposite the side with a length of 15 is $35^\circ$. Find all the possible measures of the angle opposite the side with a length of 20 to the nearest degree.

14. A triangle has two sides with lengths of 45 and 44. The measure of the angle opposite the side with a length of 44 is $62^\circ$. Find all the possible measures of the angle opposite the side with a length of 45 to the nearest degree.

15. A triangle has two sides with lengths of 63 and 75. The measure of the angle opposite the side with a length of 75 is $22^\circ$. Find all the possible measures of the angle opposite the side with a length of 63 to the nearest degree.

16. A triangle has two sides with lengths of 42 and 37. The measure of the angle opposite the latter is $20^\circ$. Find all the possible measures of the angle opposite the side with a length of 42 to the nearest degree.

17. A triangle has two sides with lengths of 15 and 9. The measure of the angle opposite the latter is $34^\circ$. How many triangles can be formed?

18. A triangle has two sides with lengths of 17 and 19. The measure of the angle opposite the latter is $5^\circ$. How many triangles can be formed?
IV. Challenge Problems

19. For the figure below find $m\angle ADB$ and $m\angle C$ to the nearest whole degree, given $m\angle ADB > m\angle C$.

20. For the figure below find $m\angle DGE$ and $m\angle F$ to the nearest whole degree, given $m\angle DGE > m\angle F$.

![Diagram](image)

21. Line segment $AB$ has a length of 15 and $m\angle A = 35^\circ$. A segment with a length of 12 will form the third side of the triangle. What are the possible measures of the angle opposite side $AB$?

22. For $\triangle ABC$, $a = 6$, $b = 10$, and $m\angle A = 42^\circ$, how many triangles can be formed?

23. For $\triangle DEF$, $e = 27$, $f = 12$, and $m\angle F = 37^\circ$. Find all possible $m\angle E$ to the nearest degree.

24. For $\triangle ABC$, $a = 15$, $b = 11$, and $m\angle B = 36^\circ$. Find all possible $m\angle C$ to the nearest degree.

25. For $\triangle DEF$, $d = 25$, $e = 30$, and $m\angle D = 40^\circ$. Find all possible measurements of $f$ to the nearest whole number.
V. Answer Key

1. \( \angle B \approx 70^\circ \) or \( 110^\circ \)
2. \( \angle C \approx 22^\circ \) or \( 158^\circ \)
3. \( \angle E \approx 70^\circ \) or \( 110^\circ \)
4. \( \angle M \approx 27^\circ \)
5. Two triangles (\( \angle B \approx 61^\circ \) or \( 119^\circ \))
6. One Triangle (\( \angle D \approx 9^\circ \))
7. \( \angle A \approx 71^\circ \) or \( 109^\circ \)
8. \( \angle N \approx 8^\circ \) or \( 172^\circ \)
9. \( \angle Z \approx 13^\circ \)
10. \( \angle A \approx 26^\circ \)
11. \( \angle F \approx 48^\circ \) or \( 132^\circ \)
12. \( \angle L \approx 53^\circ \) or \( 127^\circ \)
13. \( \approx 50^\circ \) or \( 130^\circ \)
14. \( \approx 65^\circ \) or \( 115^\circ \)
15. \( \approx 18^\circ \)
16. \( \approx 23^\circ \) or \( 157^\circ \)
17. Two triangles (\( \approx 69^\circ \) or \( 111^\circ \))
18. One Triangle (\( \approx 4^\circ \))
19. \( \angle ADB \approx 122^\circ \), \( \angle C \approx 58^\circ \)
20. \( \angle DGE \approx 107^\circ \), \( \angle F \approx 73^\circ \)
21. \( \approx 134^\circ \)
22. none, \( b \) is too short (if \( a \) is less than \( b \sin A \) no triangle is formed- \( a \) is too short)
23. there is no possible \( \angle E \) that will form a triangle with the given dimensions
24. \( \angle C \approx 17^\circ \) or \( 91^\circ \)
25. \( f \approx 7 \) or \( 39 \)