Course of Study – Advanced Algebra

1. **Introduction**: Common Core Advanced Algebra, Mathematics Department, Grade 9-12, 2 semester course.

2. **Course Description**

- Common Core Advanced Algebra is a one year advanced algebra course which covers the state of California Common Core Math State Standards (CaCCSS-M) for the second year of algebra. Over the course of the year, students will participate in a broad range of instructional experiences using a variety of methods including the Common Core mathematical practices. This class supports the District's stated mission for all students to master core competences and critically analyze information. The course is rigorous and demanding. The essential program goals are to interpret and graph polynomial functions, rational functions, exponential functions and logarithmic functions. In addition, students will master ideas of probability and single variable statistics. Successful completion of this course will prepare students for Precalculus. The intended audience is all students who are intending to continue their education at a four year college or university. All topics will be taught using Common Core Mathematical Practices:
  - **(1)** Make sense of problems and persevere in solving them.
  - **(2)** Reason abstractly and quantitatively.
  - **(3)** Construct viable arguments and critique the reasoning of others.
  - **(4)** Model with mathematics.
  - **(5)** Use appropriate tools strategically.
  - **(6)** Attend to precision.
  - **(7)** Look for and make use of structure.
  - **(8)** Look for and express regularity in repeated reasoning.

3. **Prerequisite skills** and knowledge suggested for success in the course:

- Combine algebraic expressions of all types, including adding, subtracting, and multiplying polynomials.
- Simplify algebraic expressions involving exponents, radicals, and rational expressions.
- Graph, interpret and write linear and quadratic functions both by hand and using technology.
- Solve linear and quadratic equations using a variety of methods, including factoring, the quadratic formula, and square rooting both by hand and using technology.
- Use various methods for solving systems of equations, including elimination, substitution, and graphing both by hand and using technology.
- Graph, interpret, and write equations of exponential functions both by hand and using technology.
- Graph, summarize and analyze data on a single count or measurement variable both by hand and using technology.
- Understand basic trigonometric functions and solve triangles using applications of them.

Justification: This second year Algebra course requires all mentioned prerequisite skills to maintain the rigor of a UC-Approved Advanced Algebra course.

4. List of **program goals/learning outcomes** to be met.

- **Goal-1**: Students will have the ability to analyze and graph polynomial functions.

- **Goal-2**: Students will have the ability to graph, interpret and write equations of rational functions.
- **Goal-3**: Students will have the ability to interpret, write and solve equations of exponential and logarithmic functions.
- **Goal-4**: Students will be able to calculate the probability of an event.
- **Goal-5**: Students will model, summarize, represent, and interpret data on single count or measurement variable.

5. **Learning progressions and proficiency scales** aligned with the program goals listed above.

Goal 1: Students will have the ability to analyze and graph polynomial functions.

Advanced Content (4.0): What is the more complex use of content knowledge or skill required of students to master this goal?	Students will be able to apply features of polynomial functions to real life scenarios, both with and without technology.
Target Content (3.0): What is the specific target content knowledge or skill required of students to progress towards master of this goal?	Students will be able to graph a polynomial given in standard form that has at least one rational zero. Additional zeros can be rational, irrational or imaginary. Students will be able to write the polynomial function given key features and/or a graph.
Emerging (2.0): What basic terminology, specific facts, or simple ideas are required for students to progress toward mastery of this goal?	Students will be able to determine how the degree, zeros, and leading coefficient affect the end behavior and shape of the polynomial. Students will be able to find the x and y intercepts of a polynomial function given in factored form and can use the intercepts to graph the polynomial. Given a graph, students will be able to determine the degree of the polynomial, determine whether or not the leading coefficient is positive or negative, and determine the types of zeros (real, imaginary).
Limited Content (1.0)	Students will: Demonstrate a partial understanding of some of the simpler processes and/or some of the complex processes.

Goal 2: Students will have the ability to graph, interpret and write equations of rational functions.

Advanced Content (4.0): What is the more complex use of content knowledge or skill required of students to master this goal?	Students can apply features of rational functions to real life scenarios, both with and without technology.
Target Content (3.0): What is the specific target content knowledge or skill required of students to progress towards master of this goal?	Students can accurately graph a rational function by finding the critical features including values of transformations. Graphs will be limited to those with a horizontal asymptote and one vertical asymptote.
Emerging Content (2.0): What basic terminology, specific facts, or simple ideas are required for students to progress toward mastery of this goal?	Students can perform operations on rational expressions (+/-/x/÷) and solve rational equations. Students can identify extraneous solutions. Students can find the critical features (asymptotes, intercepts, holes, and domain restrictions) of a rational function.
Limited Content (1.0)	Students will: Demonstrate a partial understanding of some of the simpler processes and/or some of the complex processes.

Goal 3: Students will have the ability to interpret, write, and solve equations of exponential and logarithmic functions.

Advanced Content (4.0): What is the more complex use of content knowledge or skill required of students to master this goal?	The student: • Students can apply features of exponential and logarithmic functions to real life scenarios, both with and without technology (e.g. Newton's Law of Cooling; comparing the magnitude of values within a logarithmic scale)
Target Content (3.0): What is the specific target content knowledge or skill required of students to progress towards master of this goal?	The student: • Identifies, writes, and solves exponential growth and decay equations in applications • Applies basic definitions and properties of logarithms and exponents to solve equations • Graph exponential and logarithmic functions showing intercepts and end behavior by hand in simple cases and using technology in more complicated cases
Emerging Content (2.0): What basic terminology, specific facts, or simple ideas are required for	The student: • Correctly applies order of operations in solving exponential and logarithmic equations

students to progress toward mastery of this goal?	• Identifies whether a function represents exponential growth or decay • Can correctly change a function between logarithmic and exponential form • Can find the inverse graphically or algebraically given either logarithmic or exponential form primarily using bases 2, 10, or e.
Limited Content (1.0)	Students will: Demonstrate a partial understanding of some of the simpler processes and/or some of the complex processes.

Goal 4: Students will be able to calculate the probability of an event.

Advanced Content (4.0): What is the more complex use of content knowledge or skill required of students to master this goal?	Students can apply features of probability to real life scenarios, both with and without technology. Students can demonstrate an understanding of the importance of conditional probability.
Target Content (3.0): What is the specific target content knowledge or skill required of students to progress towards master of this goal?	• Apply the Fundamental Counting Principle through numerical application • Calculate combinations (with applications to probability) and permutations (linear, circular, repetition, multiple object) • Determine if events are independent or dependent • Simplify factorial expressions (using "n" notation)
Emerging Content (2.0): What basic terminology, specific facts, or simple ideas are required for students to progress toward mastery of this goal?	• Apply the Fundamental Counting Principle graphically (Tree Diagrams) • Calculate combinations (basic) and permutations (linear and some repetition problems) • Calculate basic probability (Prob=Favorable/Total Outcomes) • Calculate n!
Limited Content (1.0)	Students will: Demonstrate a partial understanding of some of the simpler processes and/or some of the complex processes.

Goal 5: Students will model, summarize, represent, and interpret data on single count or measurement variable.

Advanced Content (4.0): What is the more complex use of content knowledge or skill required of students to master this goal?	Students can use technology to effectively summarize and interpret data collected from real life scenarios. Students can identify and define similarities and differences between data distributions and Normal distributions using techniques from level 3.0.
Target Content (3.0): What is the specific target content knowledge or skill required of students to progress towards master of this goal?	Students will recognize and justify why a real world data set may not lend itself to a Normal distribution. Students will evaluate real-world data and use the Empirical Rule to compare it to a Normal distribution.
Emerging Content (2.0): What basic terminology, specific facts, or simple ideas are required for students to progress toward mastery of this goal?	Students will: Create a histogram from data. Use the mean and standard deviation of a data set to compare it to a Normal distribution curve.
Limited Content (1.0)	Students will: Demonstrate a partial understanding of some of the simpler processes and/or some of the complex processes.

6. **Suggested scope and sequence of Curriculum**

Polynomial Functions:

- Recognize if a function exhibits polynomial behavior by graph, equation, table, or verbal description.
- Write a polynomial function that best represents a graph or sufficient information.
- Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing local maxima, intercepts and end behavior.
- Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
- Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Rational Functions & Expressions:

- Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- Create rational equations and use them to solve problems.

Exponential Functions:

- Prove that exponential functions grow by equal factors over equal intervals.
- Construct exponential functions, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Logarithmic Functions & Expressions:

- For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
- Graph logarithmic functions, showing intercepts and end behavior.
- Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Probability, Measurement & Data:

- Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
- Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- Understand the conditional probability of *A* given *B* as $P(A \text{ and } B)/P(B)$, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
- Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and*

compare the results.

- Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
- Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
- Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.
- Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
- Use permutations and combinations to compute probabilities of compound events and solve problems.
- Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
- Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
- Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
- Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*
- Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
- Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
- Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
- Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

7. **Suggested textbook(s), materials, equipment and resources**

No new common core books have been approved at this time.

8. **Requirements satisfied** (A-G, grad requirement)

Meets the Advanced Algebra Mathematics ("C" requirement).

9. **Appendix** to contain a sample unit of study and assessment(s)

Stage 1 – Desired Results

Established Goal(s):

Goal 1: Students will have the ability to analyze and graph polynomial functions.

Understanding(s):

Students will understand that…

•The zeros to all polynomials functions represent where they cross the x-axis.

•Maximum and minimum values of a polynomial function can be found between two zeros.

•End behavior of a polynomial is defined by its degree and leading coefficient.

•Polynomials functions are functions that can be graphed.

Essential Question(s):

- What types of situations can be modeled using polynomials?
- How would you know what a graph looks like without graphing it?

Students will know…

- The characteristics that determine the graph of a polynomial function

Students will be able to…

- How to graph a polynomial, including intercepts, maxima and minima, and end behavior

Stage 2 – Assessment Evidence

Performance Task(s):

Box Project – Create a box using conditions involving volume, surface area, and algebra (see diagram below for layout)

Other Evidence:

Class notes

White boards

Warm-ups

Homework assignments

Daily Quizzes: Operations with Polynomials, Factoring, Finding Zeros, Graphing Polynomials

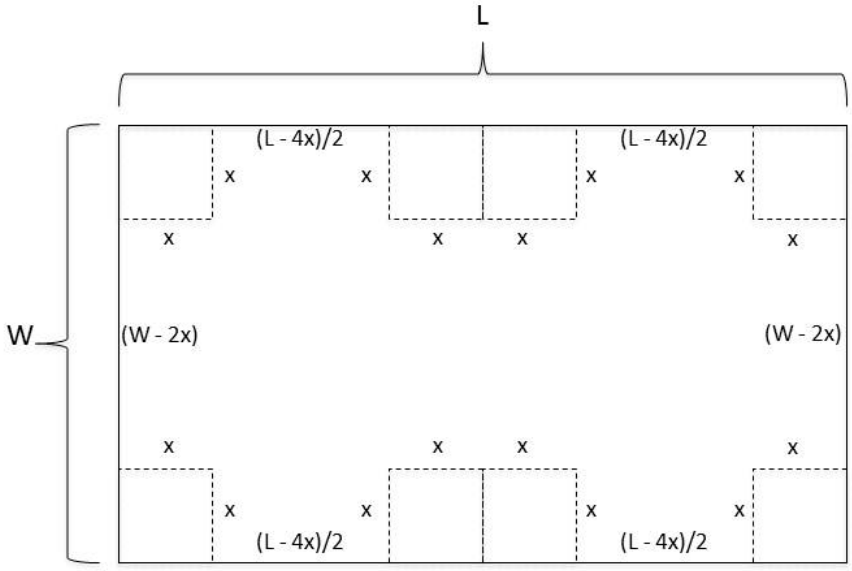
Test-Polynomial Assessment

Stage 3 – Learning Plan

Learning Activities:

3-Weeks then Unit Test

- Classification of Polynomials & Operations w/Polynomials HW from text.
- *Quiz-Operations with Polynomials*
- Characteristics and End Behavior of the graph of a Polynomial HW from text.
- Writing Polynomials as functions of an unknown HW from text.
- Solving Polynomials by factoring, long and synthetic division, substitution and HW from text.
- *Quiz-Factoring*
- Remainder & Factor Theorem HW from text HW from text.
- Finding Roots and Zeros using the Location Principal HW from text.
- *Quiz-Finding Zeros of Polynomials*
- Graphing Polynomials HW from text.
- *Quiz-Graphing Polynomials*
- *Box project*
- *Test-Polynomial Assessment*



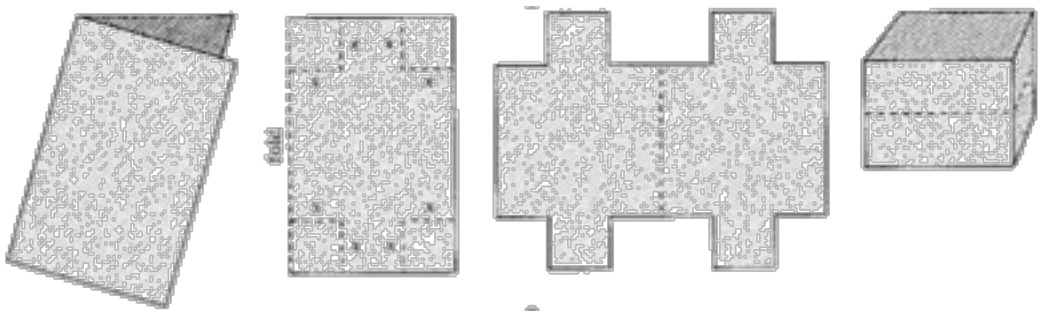
'x' is the cut-out
'L' is the long side
'W' is the short side

Polynomial Performance Task—Box Project

Advanced Algebra Due Date_____

A. Select a medium for your project. (Obtain 2 equal pieces of the medium)

ie. plexi-glass, foam board, thick poster board, wood, anything that will hold its shape.



B. Complete the tasks below (All written work should be typed. Equations can be written in.):

1) Show your calculations and graph for determining the maximum volume of a box constructed from your medium. (follow the construction diagram above)

2) Select on object of sentimental value that has a volume at least 25% less than the maximum volume of a box constructed from the medium you chose.

(Calculation the volume of your object and show that it is at least 25% less.)

3) Calculate the cut-out (x) needed to construct 2 different boxes whose volume is 10% greater than your object's volume.

4) Construct both boxes. (Boxes can be made from different media but the media must have the same starting dimensions.)

5) Find the dimensions of each box and explain your calculations.

6) Show that the volumes of your boxes are actually the same.

7) Discuss how it is possible to construct 2 different boxes with the same volume using the same sized medium.

8) Discuss the advantages and disadvantages associated with each box.

(in terms of what they could or could not be used for.)

9) If possible, place your object in one of the boxes.

10) Discuss why your object, which has a smaller volume, did or did not fit in each box.

11) Make a duty list indicating everything each group member did.

Name _____ Partner's Name_____

Phone_____ Phone _____