# TAMALPAIS UNION HIGH SCHOOL DISTRICT
**Larkspur, California**

**Course of Study and Unit Plans**

**Calculus 1-2**

## I. INTRODUCTION

Calculus is a one-year course offered to seniors who have finished Pre-Calculus and either have a schedule full of Advanced Placement classes or who are not ready for the rigor of an AP calculus course. The course will cover differential and integral calculus with an emphasis on using visuals and explorations to understand calculus concepts in a concrete manner. Students will learn to use the graphics calculator effectively; in each unit, students will perform lab experiments, gather data in a calculus context, make connections between the unit of study and the prior units, and write regularly in a journal about calculus concepts.

This course addresses the following Tam 21st Century Mission, Beliefs, and Goals in the following ways:

The learning environment provides opportunities for students to:
- Acquire, manage, and use knowledge of calculus concepts and skills (mission)
- Think critically and creatively when developing a conceptual understanding of calculus (mission)
- Practice self-directed learning, decision making, and problem solving (mission)
- Participate in a program for high achieving students (philosophy)
- Take part in the full learning experience: challenge, exploration, risk-taking, initiative, hard work, success and failure, and joy and enthusiasm (belief)

This course addresses the following Student Learning Outcomes:
Outcome 1: Communicate articulately, effectively, and persuasively when speaking and writing.
Outcome 2: Students will read and analyze material in a variety of disciplines.
Outcome 3: Use technology as a tool to access information, analyze and solve problems and communicate ideas.
Outcome 5: Apply mathematical knowledge and skills to analyze and solve problems.
This course is designed to help students attain the state subject Content Standards.

## II. STUDENT LEARNING OUTCOMES AND STATE STANDARDS

A. The overarching themes of Calculus 1-2 in which students will become accomplished are:

1. With the use of the graphics calculator, students will be able to examine graphs, tables, and equations in order to broaden their perspective and make connections between calculus concepts.

2. For each branch of calculus, students will apply the concepts of calculus in modeling problems. Students will use models that are based both on equations and data, data that is either given to students or created in experiments, in order to develop models or to analyze them.

3. Throughout the two semesters, students will use the vocabulary and notation of calculus.

B. Students will cover the following state *subject* Content Standards:

1.0 Students demonstrate knowledge of both the formal definition (*optional topic at the end of the second semester*) and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. Students know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity:

1.1 Students prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.

1.2 Students use graphical calculators to verify and estimate limits.

1.3 Students prove and use special limits, such as $\lim_{x \to 0} \frac{\sin x}{x}$ and $\lim_{x \to 0} \frac{1 - \cos x}{x}$.

2.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.

4.0 Students demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability:

4.1 Students demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function.

4.2 Students demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.

4.3 Students understand the relation between differentiability and continuity.

4.4 Students derive derivative formulas and use them to find the derivatives of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions.

5.0 Students know the chain rule and its proof and applications to the calculation of the derivative of a variety of composite functions.

7.0 Students compute derivatives of higher orders.

8.0 Students know and can apply Rolle's theorem, the Mean Value Theorem, and L'Hôpital's rule.

9.0 Students use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.

10.0 Students know Newton's method for approximating the zeros of a function.

11.0 Students use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.

12.0 Students use differentiation to solve related rate problems in a variety of pure and applied contexts.

13.0 Students know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals.

14.0 Students apply the definition of the integral to model problems in physics, economics, and so forth, obtaining results in terms of integrals.

15.0 Students demonstrate knowledge and proof of the fundamental theorem of calculus and use it to interpret integrals as antiderivatives.

16.0 Students use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work (*optional topic at the end of the second semester, if time allows*).

17.0 Students compute, by hand, the integrals of a wide variety of functions by using techniques of integration, such as substitution. They can also combine these techniques when appropriate.

18.0 Students know the definitions and properties of inverse trigonometric functions (*inverse sine and inverse tangent only*) and the expression of these functions as indefinite integrals.

20.0 Students compute the integrals of trigonometric functions by using the techniques noted above.

21.0 Students understand the algorithms involved in Simpson's rule and Newton's method. They use calculators or computers or both to approximate integrals numerically.

27.0 Students know the techniques of solution of selected elementary differential equations and their applications to a wide variety of situations, including growth-and-decay problems.

C. Calculus content is not covered in either the state Standards Tests and/or High School Exit Exam.

## III. UNITS OF INSTRUCTION

A. Major Units of Instruction

The units include the algebra of calculus, limits and continuity, the derivative and techniques of differentiation, applications and modeling using the derivative, anti-derivatives and indefinite integration, area under the curve and definite integration, and application of the definite integral and mathematical modeling.

B. Enduring Understandings/Essential Questions/Skills:

**Unit 1 – The algebra of calculus***:*
**<u>Enduring Understanding</u>***: Students will demonstrate familiarity with the properties of functions, both algebraic and trigonometric, from a traditional and graphing calculator approach, and students will also demonstrate a strong understanding of the unit circle.*

**<u>Essential Question</u>***:*
*How do equations, graphs, and characteristics of both help us to understand functions? How are they related for familiar functions?*

*How can the use of technology aid our understanding of functions?*

*How can knowledge of the unit circle aid our ability to graph and evaluate trigonometric functions?*

**Knowledge and Skills:**

Find composition of functions, derive inverses, replace (x + h) for x, recognize parent graphs, identify domain and range, graph by transformations, sketch graphs with emphasis on piecewise graphs and critical features of rational graphs, use even- and odd-ness to aid in the sketching of functions.

Simplify compound fractions (especially in the form $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{x+h}$), rewrite expressions as single fractions, remedy predictable common errors (such as T or F: $\frac{1}{p+q}=\frac{1}{p}+\frac{1}{q}$, $\frac{x+y}{2}=\frac{x}{2}+\frac{y}{2}$), rationalize numerators and denominators, simplify quotients by factoring, solve equations by clearing denominators or cross multiplying.

When working with inequalities, students use proper interval notation, use number line charts and sign charts to determine solutions; solve linear, quadratic, and rational inequalities, solve absolute value inequalities, graph absolute value functions, rewrite absolute value functions as piecewise functions.

Understand fractional and negative exponents, change from radical form to exponent form, from exponent to logarithmic and vice versa form, graph exponential, radical, and logarithmic functions, simplify logarithms, employ logarithmic properties.

Understand right triangle ratios, rewrite ratios in terms of sine and cosine, rewrite ratios on the coordinate plane in terms of x and y, know the unit circle, graph sine, cosine, tangent, secant, inverse sine, inverse tangent, use the sum and difference formulas, double angle formulas, Pythagorean identities.

Use the graphing calculator: table, lists, graphing functions, solver, set appropriate windows, zoom way in and way out, develop mathematical models from data, choose equations to fit data.

**Unit 2: Limits and continuity**

**Enduring Understanding:** *Students will analyze and identify limits of functions numerically, graphically, and analytically, and demonstrate familiarity with the definition of continuity and the types of discontinuities that exist.*

**Essential Questions:**

*How do limits serve as a means to better understand functions and their behavior?*

*What are the key attributes of continuous and discontinuous functions, and how does understanding those attributes aid in our overall understanding of functions?*

**Knowledge and Skills:**
Find limits by use of graphs, tables, and visuals; one sided limits, two sided limits, infinite limits, and cases where limits do not exist.

Computing limits techniques: direct substitution, factoring and canceling, visualizing graphs with asymptotes, rationalizing numerators, using end behavior, horizontal asymptotes, indeterminate forms of trigonometric ratios such as the $\lim_{h\to 0}\frac{\sin h}{h}$

Continuity: definition, on an interval, properties of continuous functions, continuity of composition of functions, Intermediate Value theorem, continuity of inverse functions, piecewise functions, removable vs. non-removable discontinuities (Mystery curve exploration from A Watched Cup Never Cools)

**Units 3 - 5: The Derivative, Techniques, Applications, and Modeling**

**Enduring Understandings:**
*Students will differentiate explicitly and implicitly on algebraic, trigonometric, inverse trigonometric, logarithmic, exponential, and inverse functions using techniques such as the power rule, the product rule, the quotient rule, and the chain rule.*

*Students will use the derivative of a function to write equations of tangent lines, to discuss the velocity and acceleration of an object, to identify intervals on which the function is increasing, decreasing, concave up, or concave down, to locate extrema and points of inflection, and to solve related rate and optimization problems.*

**Essential Questions:**
*What is a derivative?*

*How are derivatives used to locate relative and absolute extrema and points of inflection?*

*How does a derivative enable us to optimize a function?*

**Knowledge and Skills:** (broken down by unit)
**Unit 3: The Derivative**
Introduce the derivative through visuals by estimating slopes at points on graphs. Understand the definition of tangent line with slope *m* found through limiting

process; emphasize rate of change vocabulary; definition of slope using a difference quotient between two general points; notation; differentiability and continuity.

Definition of the derivative: variety of notation, vocabulary for derivative; using definition of the derivative to calculate slopes of tangent lines; writing equations of tangent lines; what makes $f'(a)$ exist vs. not exist.

Using derivatives to compute instantaneous velocity; comparing average velocity to instantaneous velocity; slopes of secant lines and tangent lines (given data, sketch, find derivative at given points, find average rate of change on an interval; emphasize units).

Techniques of differentiation: constant, power rule, multiplied by a constant, sums and differences, higher order derivatives with notation, product rule (Leibniz's proof of the product rule), quotient rule.

Graphing connections: functions and their derivatives (sketching f' given f).

Rates of change and modeling: projectile motion; velocity and acceleration connections; marginal cost; revenue

Trigonometric functions and their derivatives: using sum formulas to derive the derivative of sine and cosine; derivatives by graphing, patterns in trigonometric derivatives, using quotient rule to find derivative of tangent, secant; (Radians vs. degrees, same derivatives?).

**Unit 4: More Differentiation**

The chain rule; (prerequisite activity: identifying functions that make up a composed function); repeated chains; using graphs of f(x) and g(x) to find derivatives of f(g(x)) and g(f(x)); using tables of numerical values for f, f', g, g' to find derivatives of f(g(x)) and g(f(x)).

Implicit Differentiation: slopes on a circle, derivatives of curves that are not functions, higher order derivatives found implicitly.

Derivatives of exponential and logarithmic functions (review logarithmic properties and changing from exponential to logarithmic form and vice versa); use the definition of derivative to find derivative of $e^x$, use inverse relationship to find derivative of ln$x$; using logarithmic differentiation as a technique for calculating the derivative of $a^u$, for calculating functions raised to functional powers, and for simplifying complicated expressions.

Derivatives of inverse functions: with and without finding the inverse function; using one to one-ness of a function to determine if the inverse of a function is a function; do non-function inverses have derivatives? Derivatives of inverse sine and inverse tangent.

Using derivatives to evaluate limits: L'Hopital's Rule and indeterminate forms; revisit indeterminate forms of trigonometric ratios from unit 2; relative rates of growth.

**Unit 5: Applications and Modeling using the Derivative**

Related rates (prerequisite activity: review volume of solids) (How many licks? Exploration from A Watched Cup Never Cools) (Cardiac Output. Exploration from Finney Demana text).

Linear approximation and connection to equations of tangent lines; differentials; absolute change, relative change, and percent change (Exploration: Quadratic Approximation from Finney Demana text); Newton's Method (Is There No Limit to These Labs? Exploration from A Watched Cup Never Cools).

Connecting f' and f'' to f: (review using number line and sign charts) identifying where functions increase, decrease, are concave up, concave down; finding relative extrema; first derivative test, second derivative test, extreme values; verification using graphics calculator.

Sketching f given graph of f': using f' to identify where f increases, decreases, is concave up, concave down, has relative maxima; what cusps, corners, and vertical tangents of continuous functions look like on f' (Sketching f given f')

Applied maxima and minima problems (revisit skills learned in unit 1): modeling and optimization of area, volume, revenue, cost, profit, distance (A Watched Cup Never Cools exploration from the book of the same name) (Postal problem) (Prism Pop exploration from A Watched Cup Never Cools); minimal path problems: revisit minimum path problems from pre-calculus class where only way to find solution was using graphics calculator

Mean Value theorem and Rolle’s theorem (The Meaning of Mean exploration from A Watched Cup Never Cools)

Rectilinear motion: position, velocity, and acceleration and their connections; finding instantaneous and average velocity given position; motion along a vertical or horizontal line; what first and second derivative mean in terms of motion on the line; displacement and total distance traveled on an interval; using graphs of position and velocity to estimate velocity and acceleration over time and at a point, respectively; speeding up and slowing down; (A Moving Experience exploration from A Watched Cup Never Cools)

## Unit 6: Antiderivatives and Indefinite Integration

**Enduring Understandings:**

*Students will evaluate and analyze antiderivatives (indefinite integrals) of algebraic functions, use them to solve elementary differential equations, and sketch integral curves on slope fields or generate their own slope field for a given differential equation.*

*Students will develop techniques of integration.*

**Essential Questions:**

*How do slope fields help us to approximate solutions?*

*How can differential equations be used to model and solve rectilinear motion problems?*

**Knowledge and Skill:**

Integral notation and vocabulary; antidifferentiation with connection to differential forms.

Integral curves; slope fields; writing calculator programs to create slope fields; solving initial value problems by separation of variables; using slope fields to show general solutions of differential equations and particular solutions of differential equations with initial values.

Rectilinear motion given constant acceleration and initial values; projectile motion.

Techniques of antidifferentiation: (revisit working with fractions) splitting the numerator, using inverse trigonometric functions; u-substitution.

## Units 7-8: The Definite Integral, Application, and Modeling

**Enduring Understandings:**

*Students will employ rectangular and trapezoidal approximation techniques to approximate the area under a curve, to estimate the value of the definite integral, and to approximate the value the net signed and total area.*

*Students will use the first and second Fundamental Theorems of calculus with numerical and variable limits of integration.*

*Students will compute the area of a region between two curves using integration and extend this concept to finding the volume of a solid of revolution using the methods of disks, washers, cylindrical shells, and using known cross sections.*

*Students will use the definite integral for applications and modeling.*

**Essential Questions:**

*How does the concept of integration help us to find the net-signed area?*

*What is the fundamental theorem of calculus and its significance?*

*What are three applications of the definite integral?*

**Knowledge and Skill:**

**Unit 7: Area Under the Curve and Definite Integration**

Estimating area under the curve: using rectangles (left, right, midpoint methods); using lists in the calculator to refine the estimates with greater numbers of rectangles; using trapezoids; making connections between trapezoidal rule and the left and right rectangular sums; (Explore: Area under a parabolic arch); using Simpson's Rule (Square Pegs in Round Holes exploration from A Watched Cup Never Cools).

Sigma notation and Riemann sums; using lists as n goes to infinity; Riemann sums and the definite integral; notation and vocabulary; meaning of net signed area; net signed area vs. total area; displacement vs. total distance traveled on an interval.

Fundamental Theorem of calculus: parts 1 and 2; with functions as limits of integration (upper, lower, or both); properties of definite integrals; revisit techniques of antidifferentiation with substitution and changing of limits of integration; recognizing familiar geometric forms; given graphs of integrals with variable upper limit, evaluate values; integrals as net change, given equations or data.

Average value of a function: numerically and graphically; rectilinear motion revisited: finding average velocity given a) position and b) velocity (It Averages out in the End exploration from A Watched Cup Never Cools).

**Unit 8: Applications of the Definite Integral and Mathematical Modeling**

(Review exponential growth and decay in populations, compound interest problems)

Solving growth and decay differential equations in the form $\frac{dy}{dx} = ky$ and $\frac{dy}{dx} = ky(1 - \frac{y}{L})$; Newton's Law of Cooling; make connections to slope fields; Euler's Method; connections to linear approximation.

Area between curves: (review graphing toolkit and solving equations; explicit teaching of graphing equations that are not functions in the form of x (y) =…) non-intersecting regions; intersecting regions, including change of upper graphs; with respect to y (A River Runs Through It exploration from A Watched Cup Never Cools)

Volumes by slicing: disks, washers with respect to x and with respect to y; rotating about the x or y axis, rotating about a vertical or horizontal line that is not the x or y axis;

Volumes by cross sections (Home in the Dome exploration from A Watched Cup Never Cools); Volumes by cylindrical shells (As Easy as pi exploration from A Watched Cup Never Cools)

Lengths of curves; arc length of a smooth curve; area of a surface of revolution ($H_2O$ in the S-K-Y exploration from A Watched Cup Never Cools)

Optional topics at the end of the year: Formal definition of limit; Work in the direction of motion: done by a constant force; done by a variable force; fluid pressure and force; fluid density; fluid pressure; fluid force on a vertical surface; emphasis on units; hyperbolics: definitions, graphs, identities, hanging cables, derivatives of, integrals, the St. Louis Arch and an exploration of the length of it.

C. Student Assessments

*Rationale:* Assessment and instruction should be aligned and designed to promote mathematical thinking. Teachers should move towards the use of engaging problem situations that involve students in investigating, conjecturing, verifying, applying, evaluating, and communicating with their assessments.

*Multiple Measures:* Assessment should be made on the basis of a variety of means such as quizzes, tests, investigations, labs, unit summaries, and projects. Informal assessments can involve daily work and writing samples. Because students will be spending some class time learning concepts with team members, team assessments are possible. The use of student journals to show growth and progress is encouraged.

## IV. METHODS, MATERIALS, AND RESOURCES

A. Methods

Calculus will be taught using direct teaching strategies, through explorations with data (either student generated or from resources), by having students perform experiments, analyzing results, and drawing conclusions, utilizing a graphics calculator individually, in pairs, and in a large group setting that is teacher directed, having students write reflective responses or laying out explanations of concepts or statements in a journal, through visual and concrete examples, and by working in small groups of two, three, or four students.

B. Materials

Main textbook:
Calculus: Early Transcendentals, Single Variable by Howard Anton, Irl Bivens, and Stephen Davis ,2005, Wiley and Sons,

Supplemental Instructional Materials
Secondary text: Calculus: Graphical, Numerical, Algebraic by Ross Finney, Franklin Demana, Bert Waits, and Daniel Kennedy; Scott Foresman Addison Wesley, 1999 edition.

Supplemental Resources:

- The Algebra of Calculus with Trigonometry and Analytic Geometry (resource book) by Eric J. Braude, DC Heath and Co, 1990.

- A Watched Cup Never Cools (resource book) by Ellen Kamischke, Key Curriculum Press, 1999.

- Mathematics Teacher (magazine, various volumes from 1995 to current)

- 3-Dimensional wooden models for volumes by Foster Manufacturing Co, Plano, TX: Disk method, washer method, cylindrical shells, volumes by cross sections (both as solids and cut into cross sections)

C. Technology

- Class set of TI-84 Plus Silver edition graphics calculators
- Autograph Dynamic Mathematics Software from Autograph Maths (on teaching station)
- LCD projector to use with Autograph software
- Avervision Document camera for projection of the graphics calculator, to share teacher and student work

## V. ANCHORS OF STUDENT WORK

In Unit 1, The Algebra of Calculus, students explore families of functions and transformations in the calculus lab, Foundation Work. Students will become more acquainted with their graphing calculator as they investigate the effect adding a constant outside or inside the function, multiplying the function by a constant or multiplying the variable inside the function by a constant, how one finds and uses intercepts. Students will work with functions that are familiar from previous courses.

In Unit 2, Limits and Continuity, the activity Computing Limit Techniques will assist students in an application of limits in relation to population. The writing prompts make sure students not only are able to find answers, but they are able to understand their meanings in context. In the Mystery Curve lab, students will explore the meaning of continuity, differentiability, and other analytic features of curves. Students will be able to explore multiple approaches to work through the meanings. Since students will not be able to use a single polynomial, they will need to create two or more different equations that also maintain continuity and differentiability. In a journal entry, students will explore the meaning of continuity and differentiability given a graph. This visual approach will help to solidify the concept of continuity. Students also have the opportunity to write about removable points of discontinuity and what it looks like when/if they occur.

In Unit 3, The Derivative, the activity "rates of change over an interval and at a point" will allow students to work from a table and a graph. In this activity, students estimate the average and instantaneous rate of change in a quantity for a variety of situations using a table or graph. Units associated with the rate of change will also be discussed when the students share their results with the class. In a journal entry, "Using the definition of derivative to calculate slopes of tangent lines and writing equations for tangent lines," students will describe what happens to the tangent at a given point as the value of the point changes. The students will calculate slopes of curves given particular values and then extend their understanding to explaining what happens in general when $x = a$. Students will also be given a table of values to explore the meaning of derivative given actual data, including units. Students will have to sketch the derivative function using the idea of slopes of secant lines. In the Leibniz's Proof activity, students will be exposed to a proof of the product rule given by Leibniz; they will get the flavor of the politics of mathematics during his time. This proof will involve using techniques of differentiation while explaining Leibniz's proof of the product rule. In working to provide visual connections to the derivative, students will be given a set of graphs and corresponding graphs of the function f. Students will connect their understanding of the features of the function and match it to the corresponding derivative graph. In another writing exercise, students will be given one graph with three graphed functions relating to rectilinear motion; students will use the key features of functions and their derivatives and the relationship between velocity, acceleration, and position to explain which graph is which and why. The final activity in this unit is an exploration of trigonometric functions and their derivatives; students will use their graphing calculator to explore limits and derivatives of trig functions in degrees and radians. The exploration of higher order derivatives will reveal the disadvantages of the derivatives that they have obtained working in degrees.

In Unit 4, More Differentiation, students apply their understanding of the chain rule in many different contexts. In one activity, students use a table of values of two functions at a given point and the derivative function values at the same point to find derivatives of new functions that result from products, quotients, and compositions. Because students are not given function rules, they will have to distinguish when to use the product, quotient, and chain rule based on the operations of the given functions. In another activity, students are asked to look at the definition of the derivative when using the chain rule on an absolute value function and consider how continuity acts in relation to differentiable functions and functions that are not differentiable at a point. Additionally, students use the graphing calculator to discover the derivative of sin2x using the definition of the derivative. Students will be directed to use graphical transformations of sin2x and sinx to make conjectures about chains of trigonometric functions.

In Unit 5, The Derivative and Mathematical Modeling, students launch the unit with an investigation (How Many Licks) of a related rate to find a rate of change using real world imperfect data. Students will use Tootsie Pops and measure them at regular intervals to see how the volume and radius change over time. The graphics calculator will be a tool that the students use to graph and analyze their data. In the exploration Cardiac Output, students will work in pairs to understand a real-life application for related rates. The problem asks students to analyze what happens to the rate of change over time for the cardiac output. In the first exploration related to Linear approximation and the connection to equations of tangent lines (Quadratic Approximation), students will take their understanding of tangent lines and find a quadratic approximation. This exploration is worthwhile as it deepens the meaning of approximation and has them reflect on the weaknesses in the linear approximation model. It is essential to include the graphing calculator in this activity so students can see the visual connection of the approximation to the actual function. In the second calculus lab in this unit, Is there no limit to these labs?, students will be led through the discovery of L'Hopital's Rule. They examine functions that approach 0/0 as x approaches a specific value. They compare the limits numerically and graphically to determine a value for the limit. They then replace the numerator and denominator functions with linear approximations at the point in question. Students will also examine the case graphically and numerically. They examine the effect as x gets larger by looking at the slopes of the numerator and denominator. These activities will reinforce the fact that the forms 0/0 and infinity divided by infinity are indeterminate forms. In an activity connected to a graphing exploration in unit 3, students will sketch the graph of f given the graph of its derivative, f prime. Students will be given several derivative graphs and asked to synthesize and apply their understanding of critical points, relative and absolute extrema, points of inflection, concavity, increasing/decreasing, and discontinuities to sketch the function. In addition, students will be encouraged to create sign charts as a visual aid in their creation of the function. Applied maxima and minima problems will be a definite focus in unit 5. The first activity (A Watched Cup Never Cools) introduces students to derivatives of exponential functions and the graph of the derivative. The lab uses Newton's law of cooling and helps students discover that actual data does not always follow ideal formulas. Students collect data on a cooling liquid and use their graphing calculator to analyze the rate of change. They are then able to use the derivative function and graphing calculator to identify minima and maxima. A problem that comes out of the US postal regulations is designed for students to find the maximum volume of a box given certain restrictions on size given by the postal service. Students will work with packing boxes to derive a formula for the volume of a box and surface area; a necessary result is that the students will have to use substitution of one variable for another. After working with the models and developing values, the students will use the derivative to find the extreme values for the postal function. Students will be encouraged to use their graphing calculator to check the validity of their solutions. Domain in context will arise naturally and its importance will be stressed. The third activity (Prism Pop) gives students practical experience in optimization. The actual calculations are simple, but real constraints can make the problem more challenging. Students will actually look at the design feature of items such as soda cans and decide why the companies design them the way they do. The students will use the ideas of optimization to create a case of cans given different constraints related to a company and costs. Students will build models of their cans and present their work to the class. In the calculus lab, The Meaning of Mean, students will write in their journals about the Mean Value Theorem and Rolle's theorem. Students will explain the underlying concept of the Mean Value Theorem using clear and concise mathematical terminology. The connection between continuity and differentiability will be strengthened because students are asked to explain their relation to the Mean Value Theorem in an abstract way.

In Unit 6, Antiderivatives and Indefinite Integration, the first activity will be one that students do in pairs as an introduction to integration. In this very simple exploration, students will be asked to make educated guesses for an antiderivative given a function written in derivative form. In the beginning, the word antiderivative will not be used. Students will see when comparing their work that many different functions can lead to the same derivative differing only by a constant C. Students will derive the power rule for antiderivatives when their collective work is examined in a whole class discussion. At this point, the constant C will be introduced and stressed. After working on antidifferentiation techniques, students will work through an investigation on stopping a car that is moving at a constant velocity when an accident occurs. Students will use their new concept of antidifferentiation and the given constant information to find a value for C; the C value will have meaning to the students since the problem is based on an actual scenario.

In Unit 7, Area under the curve and the Definite Integral, students will work to estimate the area under the curve using rectangles and trapezoids. In the first activity (Cardiac Output revisited), students are given a data table and graph that they will use to find the area under the curve using rectangles. They will be given the freedom to determine how to draw the rectangles. Eventually, the idea of underestimate and overestimate will be discussed in relation to increasing/decreasing and concavity. In the second activity (Applications using data), students will use the trapezoid rule to find the area under the curve given a list of data. Students may find it easier to find the area if they sketch a graph to see each trapezoid. This visual will help to reinforce the idea of doubling each interior base length. In the third activity (Area under a parabolic arch), students are led through a series of steps that lead to the derivation of Simpson’s Rule. In the fourth activity (Square Pegs in Round Holes), students explore rectangular approximations of area with approximations using trapezoids and finally the parabola-topped pieces of Simpson's Method. Students will then share the accuracy of their approximations and analyze the advantages and disadvantages of the methods. Moving to the definite integral and the connections between sigma notation, Riemann Sums, and net signed-area, (Writing to learn about net sales), students are given a function with labels on the axes. They are asked to interpret what the area under the curve on a closed interval would represent in context. This is a reinforcement of applications of integration. In a second journal writing assignment, students are asked to observe and describe the connection between the definite integral and practical applications. Given the speed of a car at ten second intervals as a student travels to the market, students construct a method to estimate the distance traveled over the time period; students extend their ideas to the return trip home and how that data might look on a graph. This will lead to a discussion about the difference between total distance traveled and displacement. Explorations involving both parts of the Fundamental Theorem of Calculus will include data sets. Both of these activities are designed to assist the student in understanding the meaning of integration and its connection to a rate of change since the data set involves products and the number of products produced over time. The problems give students a chance to use regression on the calculator again. Students will make a graphical connection to the definite integral when determining the totals for the given time periods. The calculus lab, It Averages out in the End, will allow the students to explore average value of a function, both numerically and graphically. The objective of this investigation is for students to discover the formula for the average of a function and its meaning. Students will work with a variety of functions, developing tables of values, and writing a program for their calculator to determine the average height of each function. Students will use planetary data to determine the planet mean distances from the sun. Seeing the connection here helps students understand why the average value has its name.

In Unit 8, Applications of the Definite Integral and Mathematical Modeling, students will be building models and exploring objects in three dimensions to better understand the concepts of area, volume, surface area, and arc length. In the lab, A River Runs Through It, students extend their knowledge of area approximation methods to develop volume approximations. In the volumes by slicing labs, the students will be using slicing to calculate the volume inside a tent and everyday objects. The objective of the first lab (Home in Dome) is for students to apply a method for finding volumes by slicing up a real object. The most common shape the students use in the lab is the hexagon. In the second lab (As Easy as Pi), students develop the disk method for determining a volume of revolution in pairs. The final lab that we present to assist students in getting a concrete understanding of three-dimensional objects involves lengths of curves and areas of surfaces of revolution. The objective of this lab is to provide students with a real life situation in which to apply volumes and surface areas of revolution: their group is bidding on painting the town water tower. The importance of accuracy, attention to detail, and the appropriate use of units are inherent since the student groups will be submitting a bid for the painting job to the town committee. Student groups will be given information, but, as described in the instructions, some of the measurements are lacking so students are going to have to prepare a report that includes all calculations, equations, diagrams, and details to convince the committee that their bid of how much paint is needed to paint the water tower is reliable and accurate.

## VI. TROUBLE-SHOOTING GUIDE

*Teacher may continuously have to work with students on their algebra skills. Teacher may have trouble pacing the course the first time through. Students may need time to become good journal writers, group members, and problem solvers.*

## VII. COURSE ASSESSMENT

*The course will be assessed by student enrollment and interest in the class, by student success in college calculus the following year, and by the quality of work produced by the students.*

## VIII. GENERAL INFORMATION

*Calculus 1-2 is a 10-credit course open to seniors who have earned at least a C- grade in Pre-Calculus.*

A. Prerequisites

*At least a C- grade in Pre-Calculus.*

B. Requirements Met

This course may be used in partial fulfillment of the math graduation requirement.

This course is approved for the “c” requirement for UC admissions.

Board Approved: March 6, 2012