Coordinate Geometry Notes

Prerequisite Knowledge
(review these Algebra concepts before beginning your work on Coordinate Geometry).
2) Finding Slope of lines between two points.
3) Determining if two lines are parallel or perpendicular (using slope).
4) Finding the midpoint of a line segment between two points (using midpoint formula)
5) Determining the length of a segment between two points (using distance formula)

What's it all about?
Coordinate Geometry is a way to “prove” certain ideas in Geometry. It works a lot like a two column proof (in that you are limited with what we begin with, we use “for sure” steps, and prove a result that then could be used with confidence as a proof).

Basically it involves drawing the shape on a graphing grid (also called a Cartesian Coordinate System), with each vertex identified by a coordinate.

Steps in a Coordinate Geometry Proof:
1) Plotting Quadrilaterals on a Cartesian Coordinate System
   a) We will name the vertices with coordinates using either Zero or Variables. In this way the quadrilaterals we are producing are “generic” rather than specific. Thus any proofs performed will apply to ALL of those shapes rather than just one specific version.
   b) Typically we try to ensure that one of the vertices is on the origin (0, 0)
   c) Move to a vertex that is on the x axis and assign it a coordinate (0, a).
   d) Move to a coordinate that shares either x or y value with that coordinate, and assign the new value. If the distance is the same and at a right angle (square or certain kites), assign the same variable (a, a). If the distance is different, but still at a right angle, assign a new y value (a, b).
   e) Continue naming all coordinates with as few variables as possible.

2) To prove a quadrilateral is definitely a specific name, prove the definition of that name:
   a) If asked to prove it is a parallelogram, prove that it has two sets of parallel lines. To do this, find the slope of opposite sides (using Slope Formula), and confirm that slopes of opposite sides are the same.
   b) If asked to prove it is a rectangle, prove that the slopes of adjacent sides are perpendicular (which means right angles). Perpendicular slopes are opposite reciprocals (like –2 and ½ or ¾ and – 4/3 ).
   c) If asked to prove trapezoid, prove one pair of opposite sides is parallel, and the other is NOT.
   d) If asked to prove isosceles trapezoid, prove one pair of opposite sides is parallel and that the sides that are NOT parallel must be congruent (meaning the distance between the endpoints is the same for both sides…use the Distance Formula).

3) Formulas you’ll need:
   **Slope Formula**
   \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
   **Midpoint Formula**
   \[ (x, y) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]
   **Distance Formula**
   \[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
4) Sample pictures to begin with:

i) Parallelogram

\[ \begin{array}{c}
(0,0) \\
(0,b) \\
(a,0) \\
(c, b)
\end{array} \]

\[ \begin{array}{c}
(0,0) \\
(a,0) \\
(c+a, b)
\end{array} \]

ii) Rectangle

\[ \begin{array}{c}
(0,0) \\
(0,b) \\
(a,0) \\
(a, b)
\end{array} \]

iii) Trapezoid

\[ \begin{array}{c}
(0,0) \\
(a,0) \\
(c, b)
\end{array} \]

iv) Square

\[ \begin{array}{c}
(0,0) \\
(0,a) \\
(a,a)
\end{array} \]

v) Isosceles Trapezoid

\[ \begin{array}{c}
(0,0) \\
(a+c, b)
\end{array} \]

\[ \begin{array}{c}
(a,0) \\
(a, a)
\end{array} \]

\[ \begin{array}{c}
(0,0) \\
(a,0) \\
(a+b, 0)
\end{array} \]

5) Examples (with links to answers):

a) Prove that quadrilateral \( A(1,2), B(2,5), C(5,7) \) and \( D(4,4) \) is a parallelogram \textit{by using slopes}.
   Answer: http://regentsprep.org/Regents/math/geometry/GCG4/anspf1.htm

b) Prove that \( A(1,1), B(4,4), C(6,2) \) are the vertices of a right triangle.

c) Prove that quadrilateral \( A(1,-2), B(13,4), C(6,8) \) and \( D(-2,4) \) is a trapezoid, but is \textit{NOT} an isosceles trapezoid.

d) Prove that the midpoints of a parallelogram’s two diagonals meet in the same point.

e) Prove that \( A(-3,2), B(-2,6), C(2,7) \) and \( D(1,3) \) is a rhombus.
   Answer: http://regentsprep.org/Regents/math/geometry/GCG4/anspf5.htm

f) Prove that \( A(4,-1), B(5,6), C(1,3) \) is an isosceles right triangle.

g) Guinevere and Lancelot see a drawing of quadrilateral \( ABCD, \ A(2,2), B(5,-2), C(9,1) \) and \( D(6,5) \). Guinevere says the figure is a rhombus, but not a square. Lancelot says the figure is a square. Write a proof to show who is making the correct observation.
### 6) Additional Practice

Are the following lines parallel, perpendicular or neither?

<table>
<thead>
<tr>
<th>Line AB</th>
<th>Line CD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8, 5) and (-6, -3)</td>
<td>(0, -7) and (2, 5)</td>
</tr>
<tr>
<td>(3, -5) and (-29, 51)</td>
<td>(-2, 7) and (0, -2)</td>
</tr>
<tr>
<td>(0, 6) and (2, -2)</td>
<td>(-4, -1) and (7, -8)</td>
</tr>
<tr>
<td>(-9, -3) and (-7, -11)</td>
<td>(8, 9) and (-13, 42)</td>
</tr>
<tr>
<td>(1, -3) and (-7, -2)</td>
<td>(6, 5) and (5, 1)</td>
</tr>
<tr>
<td>(-1, 5) and (2, -19)</td>
<td>(4, -1) and (-12, 3)</td>
</tr>
<tr>
<td>(9, -7) and (16, 0)</td>
<td>(9, -7) and (1, -8)</td>
</tr>
<tr>
<td>(6, 7) and (8, 6)</td>
<td>(11, -5) and (3, -6)</td>
</tr>
</tbody>
</table>

Find the missing Coordinate to form collinear points.

1. (-11, 7), (-16, -9), and (?, -57)
2. (-16, -1), (18, ?), and (-84, 17)
3. (-8, ?), (2, 10), and (32, 58)
4. (?, -1), (7, -4), and (31, -7)

Use Coordinate Geometry to: a) Determine if ABCD is a parallelogram. If so, b) Determine if ABCD is a rectangle.

1. (3, -2), (-2, -2), (-2, -7), and (3, -7)
2. (1, 0), (4, 0), (4, -3), and (1, -8)
3. (-3, 3), (2, 3), (5, 7), and (0, 7)
4. (2, 3), (7, 3), (7, -3), and (2, -3)
5. (-3, -7), (6, -7), (6, 2), and (-3, 2)
6. (-5, 0), (3, 0), (3, -9), and (-5, -9)
7. (-1, -4), (-4, 2), (-4, 6), and (-1, 6)
8. (3, -6), (9, -6), (7, -1), and (1, -1)
Use Coordinate Geometry to Determine if the points determine the shape indicated. (ie. prove that it is or isn’t by using slope of the sides)

1. Given points:
   (-11, -5), (-15, -3), (-15, -13), and (-19, -11).
   Is this quadrilateral a rectangle?

2. Given points:
   (-13, -12), (-4, -3), and (5, -18).
   Is this a right triangle?

3. Given points:
   (2, -10), (-4, -19), (4, -4), and (-2, -13).
   Is this quadrilateral a parallelogram?

4. Given points:
   (3, -6), (5, -11), (-7, -10), and (-5, -15).
   Is this quadrilateral a rectangle?

5. Given points:
   (-11, -15), (4, -3), and (-27, 5).
   Is this a right triangle?

6. Given points:
   (-17, 56), (-21, 55), (-10, 65), and (-14, 64).
   Is this quadrilateral a parallelogram?

7. Given points:
   (22, 14), (-18, 30), (30, 34), and (-10, 50).
   Is this quadrilateral a rectangle?

8. Given points:
   (71, -69), (66, -65), (69, -74), and (64, -72).
   Is this quadrilateral a square?