Pull

ZAP

Normal Person

I guess I shouldn't do that

Scientist

I wonder if that happens every time.
“Of what use is a child?”
Michael Faraday (when asked of what use was his discovery of electromagnetic induction)

SECTION 3
ELECTRICITY

FOR THOUSANDS OF years people have been intrigued by the effects of electricity. The word “electron” comes from the Greek word for amber (fossilized tree sap). People noticed that when amber was rubbed against some animal fur or fabric it could then be used to attract little bits of dust and matter. Where the invisible force came from to create such a thing was a deep mystery. It still is to most people, and we amuse ourselves in the same way as the ancient Greeks. It’s a rare birthday party where you don’t find someone rubbing his hair with a balloon so that he can stick it to the wall. That kind of electrical phenomenon is due to static electricity (excess electric charges that don’t move). You’ll understand later that these excess stored charges all have energy, so static electricity can be thought of as static or stored energy. Too much static charge buildup will cause a static discharge, like the kind you observe if you’ve shuffled your feet against the carpet and then reach for a doorknob (ouch!). You feel that stored energy. Lightning will strike for exactly the same reason. But it’s on a much grander scale with a much larger discharge distance and a much larger release of energy.
You can think of the static discharge as a type of **current electricity** (electric charges on the move). And if these energy-laden charges are moving, then it’s really energy on the move (although in the case of a static discharge, the current is a transient one that exists only for a moment). The kind of electricity used in the circuits of electrical devices and the kind of electricity provided by electric power companies can be thought of as a continuous current. At the end of the 19th Century, electricity was far from being just a curious phenomenon. Physicists (and capitalists) began to understand that electricity was USEFUL!

The beginning of the twentieth century ushered in a new world – a world dominated by and increasingly dependent upon electricity. We cannot conceive of living without electricity (think of the anxiety that the threat of rolling blackouts produced a few years ago). To do so would mean living in a world without microwave ovens and cell phones and DVD players and cars and McDonald’s and computers and TVs and automatic sprinkler systems and running water and airplanes and ... light (without creating a fire). That’s the worst. That’s one aspect of our humanness that profoundly separates us from all other creatures – the ability to make day out of night. To understand electricity is to appreciate the quintessential element that defines modern culture.
CHAPTER 7:
STATIC ELECTRICITY

ELEKTRON IS THE Greek word for amber. The Greeks noticed that when this fossilized tree sap was rubbed on fur it would mysteriously attract bits of dust or bits of matter. You can do the same sort of thing just by running a comb through your hair on a reasonably dry day. The comb will pick up little bits of torn up paper or attract your hair if you bring the comb close again. The comb will even attract a thin stream of water flowing from a faucet. This is all because ... there is a thing called charge.

Now if you rub two glass rods with some silk and two rubber rods with wool, the glass rods will attract the rubber rods, but the two glass rods will repel each other. The rubber rods will repel each other too. The logical explanation for these observations is that there are two types of charge (the glass gets one type and the rubber gets the other). More than that, similar charges must repel each other and opposite charges must attract each other. This is probably familiar already, but what you may not know is that charged objects (of either type) will attract uncharged objects. Before this will make any sense, you have to understand a bit about charge and the mechanisms by which things get “charged.”

DETAILS ABOUT CHARGE

We know now that there are indeed two types of charge. The charge on the electron is arbitrarily designated as “negative” and the charge on the proton is “positive.” Although the mass of the proton is about 2,000 times greater than the mass of the electron, the negative and positive charges are identical in size. This means that in the typical atom, with equal numbers of protons and electrons, the overall charge is zero – not because there are no charges, but because there are equal numbers of opposite charges. In order for something to be charged, it must have a surplus of one type of charge. Before we discuss the details of how you get that surplus, let’s look at some facts about charge:

- In solids, positive charges do not move.
- In solids, negative charges can move
  - In conductors (typically metals) negative charges are free to move throughout the material.
  - In insulators negative charges can only move about the atom.
- In fluids, both positive and negative charges are free to move.
- Grounding is the process of connecting a charged object to the Earth in order to allow the flow of electrons into or out of the object so that it becomes neutral.

TRANSFERRING CHARGE BY CONTACT

One method of charging an object is to simply put it in contact with an object of a different substance. We’ve already talked about the ancient Greeks charging up amber by rubbing it on fur, or someone charging a comb by running it through the hair. What may not be clear is that when the amber is rubbed on the fur, the fur gets charged too. And when the comb is pulled through their hair, the hair becomes charged as well. When the comb is pulled through the hair, the hair and the comb are in close contact.
contact — so close in fact that electrons in both materials are attracted by the atoms of both substances. Each atom has a certain attraction for electrons (see Table 7.1). If the atoms in the comb have a stronger attraction for electrons than those in the hair, electrons will migrate over to the comb (see Figure 7.2). This leaves the comb negatively charged because it gained electrons. It also leaves the hair positively charged by the same amount, not because it gained positive charges, but because it lost some of its negative charges, giving the illusion of gaining positive charge. Now the comb and hair will attract each other because they are oppositely charged. But the comb will also now attract little uncharged bits of paper. This is due to a phenomenon known as **induced polarity** and can be understood by using the charge rules (stated previously).

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**Table 7.1: The Triboelectric Series**

<table>
<thead>
<tr>
<th>Material</th>
<th>Polarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rabbit Fur</td>
<td>+</td>
</tr>
<tr>
<td>Acetate</td>
<td>+</td>
</tr>
<tr>
<td>Glass</td>
<td>-</td>
</tr>
<tr>
<td>Human Hair</td>
<td>+</td>
</tr>
<tr>
<td>Nylon</td>
<td>-</td>
</tr>
<tr>
<td>Lead</td>
<td>-</td>
</tr>
<tr>
<td>Aluminum</td>
<td>-</td>
</tr>
<tr>
<td>Paper</td>
<td>-</td>
</tr>
<tr>
<td>Cotton</td>
<td>-</td>
</tr>
<tr>
<td>Wood</td>
<td>-</td>
</tr>
<tr>
<td>Hard Rubber</td>
<td>-</td>
</tr>
<tr>
<td>Mylar</td>
<td>-</td>
</tr>
<tr>
<td>Epoxy Glass</td>
<td>-</td>
</tr>
<tr>
<td>Copper</td>
<td>+</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>-</td>
</tr>
<tr>
<td>Polyester</td>
<td>-</td>
</tr>
<tr>
<td>PVC</td>
<td>-</td>
</tr>
<tr>
<td>Teflon</td>
<td>-</td>
</tr>
<tr>
<td>Silicon Rubber</td>
<td>-</td>
</tr>
</tbody>
</table>

**Figure 7.2:** The two materials have a different attraction for electrons. The top substance has a stronger attraction, so that when the two are put in contact with each other, electrons migrate to the top substance. When separated, the top substance is now negatively charged because it has gained electrons and the bottom substance is positively charged because it has lost electrons.
TRANSFERRING CHARGE BY CONDUCTION

A second method to transfer charge is by conduction. This method requires that the object being charged and the object that is the “charger” both be conductors. Let’s say that the charger is negatively charged. It repels electrons in the object being charged making it appear positively charged near the charger. When the charger makes contact, electrons move into the object being charged, charging it negatively. The following characteristics are always part of charging by conduction:

- Both the charger and the object being charged must be conductors. **Why is this?**

- Contact must be made between the charger and the object being charged. **Why is this?**

- Charging by conduction always transfers the same type of charge as the charger. **Why is this?**

Figure 7.3: Charging by conduction. A negatively charged conducting rod approaches a neutral conductor. It repels electrons in the conductor, making it appear positively charged near the charged rod. When the charged rod makes contact, electrons move into the conductor, charging it negatively.
TRANSFERRING CHARGE BY INDUCTION

A final method to transfer charge is by induction. Unlike charging by conduction, the charger does not need to be a conductor (although the object being charged must still be a conductor). Let’s say that the charger is again charged negatively. It repels electrons in the object being charged making it appear positively charged near the charger (similar to charging by conduction). However, the object being charged is now “grounded.” Grounding provides a conducting path from a conductor to the earth so that electrons can flow if influenced to do so. So, while the negatively charged charger is held close, the object being charged is touched providing a path for repelled electrons to flow out and to the ground. It is important at this point to keep the charger close before removing the ground (otherwise the electrons that were originally repelled would flow back into the object being charged). After the ground is removed, the charger can then be moved away. Since electrons were repelled out of the object being charged, it is left positively charged (opposite of the charger). The following characteristics are always part of charging by induction:

- Only the object being charged must be a conductor. **Why is this?**

- Contact is not made between the charger and the object being charged. However, if the charger is an insulator, contact may be made. **Why is this?**

- Charging by induction always transfers the opposite type of charge as the charger. **Why is this?**

Figure 7.4: Charging by induction. A negatively charged rod (conductor or insulator) repels electrons in a conductor. However, the conductor is now “grounded,” so that electrons flow out of the conductor through the ground. The charged rod is held in position and the ground is removed first. After ground is removed, the charger can then be moved away. Since electrons were repelled out of the object being charged, it is left positively charged (opposite of the charger).
**INDUCED POLARITY**

The charged comb picking up the uncharged bits of paper occurs for the same reason that the balloon rubbed on hair will stick to a wall. Both the comb and the balloon are creating an *induced polarity*. Let’s say the balloon is negatively charged. When it is brought close to the wall (assumed to be an insulator) it affects the charges in the wall. It attracts the positive charges in the wall, but they can’t move (because positive charges are immobile in solids). The balloon repels the negative charges in the wall, but because the wall is an insulator, these negative charges can’t migrate very far. But they can at least go to the far side of the atoms in the face of the wall. This makes the wall *appear* positively charged, and thus attract the negatively charged balloon.

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**Figure 7.5**: Induced Polarity. This series of three drawings illustrates why charged and uncharged objects attract each other.

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**Figure 7.6**: Induced polarity is evident in both situations pictured here. In the photograph on the left (taken by Alison Haroff, Class of 2006), neutral salt and pepper particles stick to a charged plastic spoon. In the photograph on the right (taken by Lauren Wahlstrom, Class of 2005) neutral Styrofoam packing pieces stick to a charged and inflated rubber glove.
The forces charges exert on each other

The electric force is a very, very strong force – stronger than the gravitational force by a factor of $10^{39}$. This unfathomable number is greater than a trillion times a trillion times a trillion. Figure 7.7 shows some bits of paper being attracted to a charged comb. In order for the paper to lift up off the table and attach to the comb the electric force must overcome the gravitational force that the Earth exerts on the paper. It should give you a bit of a pause to consider that the few extra charges on the comb can attract the bits of paper with greater force than the gravitational force exerted by the mass of the entire Earth. So why don’t we see the evidence of this inconceivably large force? Well, we do. We see it, for example, in the photograph of the comb and the bits of paper. The miniscule comb easily wins its tug-of-war with the Earth. You see it too when a balloon, after being rubbed on the head will stick to a wall or ceiling, easily defying the Earth’s gravitational attraction. However, the evidence for the effect of the Earth’s gravity is generally much more obvious. This though is simply because most objects have equal numbers of each kind of charge, making the net charge zero. The two kinds of charge normally cancel each other out. It’s only when more of one type of charge is found on some object that we see that the electric force makes the gravitational force less than puny.

Coulomb’s Law

Charles Coulomb was a brilliant French engineer who lived between 1736 and 1806. He decided later in his career to concentrate on physics and his greatest legacy was the understanding of how electric charges affect each other. The quantification of this understanding came to be known as Coulomb’s Law. In Coulomb’s experiment, (using an apparatus similar to the modern torsional balance shown in Figure 7.8), he started with two charged metal balls placed close to each other. Coulomb knew the quantity of charge on each ball and was able to easily measure the distance between the two balls. Coulomb was able to show that the electric force between two charged objects depended on two things: the quantity of charge on each object and the distance between the two objects. Specifically, he found that the electrical force is proportional to the product of the charges on the two objects and inversely proportional to the square of the distance between the two charged objects:

$$F_E \propto Q_1 Q_2 \quad \text{and} \quad F_E \propto \frac{1}{d^2}$$

Figure 7.7: A comb charged by running it through the author’s hair now attracts uncharged bits of paper. The phenomenon is due to induced polarity. More striking is that the small amount of excess charge on the comb exerts a greater force on the paper than the gravitational force exerted by the mass of the entire Earth.

Figure 7.8: A modern apparatus (manufactured by PASCO Scientific) used to measure the force between two charges, $Q_1$ and $Q_2$. 
This leads to the equation for Coulomb’s Law:

\[ \begin{align*}
F_E &= \frac{kQ_1Q_2}{d^2} \\
F_E &= \text{Electric force (measured in Newtons, N)} \\
Q_1, Q_2 &= \text{Quantity of charge (measured in Coulombs, C)} \\
d &= \text{Distance between the charge centers (measured in meters, m)} \\
k &= 9.0 \times 10^9 \frac{N \cdot m^2}{C^2}
\end{align*} \]

The Newton is a measure of force that gives a push or pull of about one-quarter pound. The Coulomb is a measure of charge equal to \(6.25 \times 10^{18}\) positive or negative charges.

**Example**

Two charged spheres, one with a charge of -0.16 \(\mu\)C and one with a charge of .32 \(\mu\)C, are separated by 50 cm. What force do they exert on each other?

**Solution:**

- Identify all givens (explicit and implicit) and label with the proper symbol.
  - The distance, \(d\), is given explicitly as 50 cm, but must be changed to meters before being inserted into the equation.
  - The charges are also given explicitly and can be defined arbitrarily as \(Q_1\) and \(Q_2\), but they must be converted to Coulombs before being inserted into the equation and they must be distinguished as either positive or negative.

\[ \begin{align*}
Q_1 &= -0.16 \, \mu C = -1.6 \times 10^{-8} C \\
Q_2 &= 0.32 \, \mu C = 3.2 \times 10^{-8} C \\
\end{align*} \]

**Given:**

\[ \begin{align*}
d &= 0.50 \text{ m} \\
Q_1 &= -1.6 \times 10^{-7} C \\
Q_2 &= 3.2 \times 10^{-7} C \\
\end{align*} \]

- Determine what you’re trying to find.
  - The problem asks explicitly for the force between the two charges.

\[ \begin{align*}
F_E &= \frac{kQ_1Q_2}{d^2} \\
F_E &= \frac{9 \times 10^9 \frac{N \cdot m^2}{C^2} (-1.6 \times 10^{-7} C)(3.2 \times 10^{-7} C)}{(0.50 m)^2} \\
F_E &= -0.0018 \text{ N}
\end{align*} \]

(Note the negative force indicates that the force is attractive.)

The amount of force in the example above is very small – so small that you might be tempted to think that the electric force is small in general. However, the electric force is actually extraordinarily strong. We generally think of gravity as being a strong force. After all, the world record for the high jump is only 2.45 m. So, gravity keeps even world-class athletes from getting more than about 8 feet off the ground by simply jumping away from it. But compared to the electric force, gravity is puny. Consider the forces acting between two protons. They have mass, so the two will attract each other gravitationally. But because they also have charge, they will repel each other electrically. You might think that these two opposing forces would war against each other. They do oppose each other, but the electric force easily wins. In the case of the protons, the electric force is \(10^{39}\) times greater than the gravitational force!

Take another look at Figure 7.5 to appreciate this huge difference in force. The electric force from the little bit of excess charge on the comb is able to easily lift bits of paper off a tabletop even while the gravitational force caused by the entire mass of the Earth tries to hold the paper down. The reason that this incredibly large electric force is not usually noticed is not because objects don’t have charge (they do, and lots of it). It’s because the numbers of positive and negative charges in most objects are about equal. This means that for every repulsive force between two positive charges, there will be an attractive force between a positive and negative charge. So, all the attractive and repulsive forces tend to cancel each other out.
THINKING ABOUT COULOMB’S LAW MORE CONCEPTUALLY

Most people can use Coulomb’s Law to get the right answer. However, if you ask them what it means they struggle with the concept. Coulomb’s law implies the following:

\[ F_e \propto Q_1, \quad F_e \propto Q_2 \quad \text{and} \quad F_e \propto \frac{1}{d^2} \]

This means that the electric force is proportional to the size of either of the charges and inversely proportional to the distance between the charges. That means that if either of the charges is doubled, the force between them will be doubled too. It also means that as the distance between the charges grows the force between them decreases (as you would expect), but not in the way you might expect (twice the distance \( \rightarrow \) half the force). The inverse square law requires that if the change in distance is twice as far, the effect is that the force is reduced by \( \frac{1}{4} \).

The inverse square law is difficult for many who do not have natural math talents. When you try to make predictions for changes based on the inverse square law, follow this strategy:

Let’s say that the change in distance between two charges is from 1.3 m to 2.4 m.

- Determine the factor change in distance (in this case, going from 1.3 m to 2.4 m is a factor difference of \( \frac{2.4}{1.3} = 1.85 \) times greater).
- Divide the old force by the square of the factor change (in this case, \( \frac{1}{(1.85)^2} = 0.29 \)).
- The increase in distance would cause the force in this case to be reduced by a factor of 0.29.

Let’s put both the charge effect and the distance effect together now in one problem. Let’s say that there are two charges that are 0.5 m apart and they exert a 2.0 N force on each other. What happens if the charge of one is doubled, the charge of the other is decreased to one-third of its original charge and the distance is reduced to 0.3 m?

- Charge #1 change effect
  \( Q_1 \rightarrow 2Q_1 \quad F \rightarrow 2F \)
- Charge #2 change effect
  \( Q_2 \rightarrow \frac{1}{3}Q_2 \quad F \rightarrow \frac{1}{3}F \)
- Distance change effect
  \( d \rightarrow \frac{0.3}{0.5} = \frac{3}{5} \ d \)
  \( F \rightarrow \frac{1}{(\frac{3}{5})^2} \ F = \frac{25}{9} \ F \)

Now we’ll put all the force changes together:

New force = (old force)(changes)

\( (2.0 \text{ N})(2)\left(\frac{1}{3}\right)\left(\frac{25}{9}\right) = 3.7 \text{ N} \)
ALEXIS CANFORA WAS burned on over 50% of her body on March 16, 1999. While her mother went in to pay for $5.00 worth of gas, the nine-year-old girl slid out of the back seat of her family’s 1995 Camaro and began to pump gas into the car. It’s almost certain that as she slid off the seat of the car she picked up some excess charge and then when she moved the gas nozzle closer to the gas tank opening, there was a static discharge, igniting the flowing fuel. Her instinct was to pull the nozzle from the tank, but when she did, the gasoline flowing from the nozzle at eight gallons per minute became a flame-throwing torch, which engulfed both herself and the car in a frightening inferno. Her story is not unique. You can read more about this occasional occurrence and the ongoing research to reduce such incidents at: http://www.esdjournal.com. The static discharge occurred because of the electric field which Alexis had surrounding her as she pumped the gas. In order to understand the details of how this static discharge occurred and how she could have avoided the incident, an understanding of electric fields is necessary.

Figure 7.9: A spark discharge between a gas nozzle and the filling hole of its gas tank caused an inferno that destroyed this car and severely burned the nine-year-old girl pumping gas into the car.

Figure 7.10: The electric field from the “violet ray” (on the right) excites the atoms in the fluorescent light bulb, causing it to light up. (Photo by Megan Orlando, Class of 2007.)

VISUALIZING THE ELECTRIC FIELD

All charged objects produce electric fields, and these electric fields are felt by other charges. As you know, like charges repel each other and opposite charges attract each other. They are able to communicate their presence by means of this electric field (Figure 7.10).

You can’t see the electric field, but you can easily visualize it. Imagine that you have two spheres, one positively charged and the other negatively charged. Bringing a third charge – a positive one – close to the two spheres would cause repulsion from the positive sphere and an attraction from the negative sphere. Now imagine drawing lines to represent the paths that this third charge would take if it were free to move near the two spheres. These lines are called electric field lines. Figure 7.12 shows these electric field lines around each of the charged spheres. When many field lines are drawn, the combination of all of them represents the electric field.

Electric fields are easy to draw even when there is more than just one charged object present. Here are the guidelines for constructing electric field lines:
• Field lines always point in the direction that a positive test charge would move.
• Field lines always intersect charged objects at right angles.
• Field lines never intersect each other.
• The closer field lines are to each other, the greater the electric field.

In the diagram below, a positively charged sphere is adjacent to a negatively charged sphere. A number of “test” charges have been indicated. Try to use the rules above to draw some smoothly curved field lines that would exist around these charged spheres.

Now it’s time to understand the reason for static discharge. Think about the times this may have happened to you. You’ve probably scuffed your shoes across a carpeted floor and received a small shock when you reached for a doorknob. The first step in this static discharge process is the accumulation of charge as you scuff your shoes across the carpeting. In the same way that the comb transfers charge as it is pulled through hair, the scuffing transfers charge between the carpet and you. Let’s say you pick up excess negative charge. These negative charges spread themselves as far away as they can from all the other excess negative charges, causing their distribution all over the surface of your body. This creates an electric field all around your body. Now as you reach for the doorknob, the electric field around your body is felt more strongly by the charges in the doorknob the closer you get. Electrons in the doorknob are repelled giving the surface of the doorknob a positive charge. Because of Coulomb’s Law, these positive charges in the surface of the doorknob pull more and more forcefully on the excess negative charges on your hand as it moves closer. Air is normally a good insulator, but as the electric field grows to a critically high level, it is able to rip apart the air molecules between your hand and the doorknob. These ions (free positive and negative charges) then provide a conducting path for the excess electrons in your hand to move to the doorknob. The
accompanying release of energy gives the “shock” you feel and the air molecules recombining release energy in the form of sound and light, creating the pop you hear and the mini bolt of lightning you see if it’s dark. This is probably the scenario that played out in the Alexis Canfora incident. Her shoes were apparently good insulators, preventing any charge buildup to leak to the ground. And she obviously didn’t touch anything metal, like the side of the car or the gasoline pump, before beginning to pump the gas. These methods for removing excess charge usually occur without you ever having to think about it, but the occasional occurrence of these gas station static discharges has led to some action on the part of gas companies. Electrically conductive stickers (Figure 7.13) are being seen more frequently on the sides of gas pumps, urging people to touch the side of the metal tank, and thereby ground themselves before beginning the fueling process.

Figure 7.13: Stickers like this on the sides of gas station pumps encourage customers to discharge themselves of any accumulated charge before starting to pump fuel. Otherwise, accumulated charge could cause a static discharge, possibly igniting the fuel.

CHARGE DISTRIBUTION ON CONDUCTORS
Imagine a perfectly spherical conductor with eight excess negative charges. Since it’s a conductor, the charges can move and since they’re similar charges, they will move to be as far from each other as possible. The configuration will end up looking like Figure 7.14, with the excess charges residing on the surface and all being the same distance from the nearest neighbor. This is very reasonable, but it leads to three profound ideas concerning charged conductors. The first is that the excess charge on a conductor will always reside on the surface of the conductor. This is true of course because of the repulsive force between the similar charges. This means that even if there were a very high charge on a conductor that you were inside of, you could touch the inside surface and be perfectly safe.

Figure 7.14: This conductor has an excess of negative charges. Since it’s a conductor, the charges can move and since they’re similar charges, they will move to be as far from each other as possible. This leads to all the excess charge residing on the surface of the conductor.

The second profound idea that comes from the way the charges orient themselves on the conductor is that the electric field inside a conductor will always be zero. This is more obvious with the spherical conductor shown. If you place a negative charge at the very center of the sphere, all the repulsive forces from the negative charges on the surface will cancel each other and the net force felt by the charge at the center will be zero. But if the force on the charge in the center is zero, there can’t be an electric field at that point. If there were an electric field it would move the charge at the center, but we know that won’t happen. So you may be thinking that that is just a special case and that if the charge at the center were moved, it wouldn’t feel equal forces from charges on the surface. You’re right! However, the net effect would still be the same. Figure 7.15 shows a case where a negative charge is in a location other than the center. Notice that although the forces of the closer charges on the surface are greater, there are a greater number of opposing forces acting in the opposite directions. The net effect is that the charge inside the sphere still feels a zero net force, indicating zero electric field at that point. I understand that this is hardly a rigorous proof, but perhaps you can see that conceptually this could be possible. So, without a formal mathematical
proof, let me make a suggestion that should convince you that the electric field inside a conductor must be zero. Let’s play the “devil’s advocate” and suppose that the placement of the charge on the conductor leads to an electric field inside the sphere that is not zero. That would mean that the charges on the surface would move under the influence of this field. But, since we know that they don’t move, but instead find a place of stability, the electric field inside MUST BE ZERO.

Figure 7.15: A charge is closer to one side of the inside of the sphere than when it was in the center. Notice that although the forces of the closer charges on the surface are greater, there are a greater number of opposing forces acting in the opposite directions. The net effect is that the charge inside the sphere still feels a zero net force, indicating zero electric field at that point.

In Figure 7.16, Peter Terren dramatically illustrates the phenomenon of zero electric field inside conductors … on himself! To prepare for the 500,000-volt discharge seen in the photograph, he wrapped most of his body in foil (underneath his clothing). There is also a fine wire mesh mask covering his face. The body-shaped conductor gives him absolute safety from the high voltage discharge because he is inside of it. All the charge stays on the outside of the foil suit leaking to the ground off the area around his right foot. Obviously, being a spherical shape is not a requirement for this phenomenon.

So, no matter the shape of the conductor, the electric field inside will always be zero. This leads to the third profound idea: **charges on conductors will concentrate at the pointy parts of the conductor** (if there are any). This is illustrated in Figure 7.17. Since the pointy parts of the conductor will generally be farther from a charge inside the conductor than the smoother parts, more charges must accumulate at the pointy parts so that the electric field will be zero inside (something we know must be the case).

Figure 7.16: Inventor Peter Terren is protected from the dangers of a 500,000-volt static electric shock because his body is completely enclosed within a conductor. Under his clothing, a body suit of metal foil and a fine metal mesh mask over his face exclude any electric field on or in his body. The charge (from a large Tesla coil, moved in an arc over his body) stays on the outside of the conducting body suit and leaks to the ground off the area around his right foot.

Figure 7.17: Since the pointy parts of the conductor will generally be farther from a charge inside the conductor than the smoother parts, more charges must accumulate at the pointy parts so that the electric field will be zero inside (something we know must be the case).
This condition of charges concentrating at the pointy parts of conductors leads to static discharges being common in those regions. Since the charges accumulate at the pointy parts, the electric field is also strongest at these points. That means that if the ripping apart of air molecules and the resulting spark discharge takes place because some conductor has excess charge built up on it, it will happen where the conductor is pointiest. You’ve probably experienced this as you’ve walked across a carpeted floor, accumulating charge by contact. You reach for a doorknob and … zap! The spark discharge (and the shock that you feel as a result) occurs at the pointiest part of you – your fingers. It’s not always bad though. If you live in a place where lightning is common, you can put a lightning rod on your house (see Figure 7.18), making it the pointiest part of the house. Then, if a thunderstorm occurs and the electric field builds between the house and the clouds, spark discharges will start to take place at the lightning rod, removing the charge from the house and therefore preventing the lightning from striking.

**Electric Potential Energy**

So far in this discussion of the electric field, we’ve looked at static discharge, electric shielding, and distribution of charge on conductors, but the importance of the electric field for flowing electricity is the electric potential energy that the field gives to charges in an electric circuit. A good way to begin to understand how the electric field gives potential energy to charges is to first consider how the gravitational field gives potential energy to masses. Figure 7.19 shows the electric field around a negatively charged sphere (on the left) and the gravitational field around a spherically shaped mass (on the right). You should notice that the two fields are identical. The only difference is that the electric field causes forces on charges, but the gravitational field causes forces on masses.

The reason for considering the gravitational field here is that … you get it! You know that it’s hard to run up a long flight of stairs. Thinking about this in terms of the gravitational field, it’s hard to run up that flight of stairs because you’re going against the field – the field lines all point toward the center of the Earth. The farther you go against the gravitational field (away from the Earth), the more potential energy you store in your body. That’s why it’s scarier to jump off the high dive than the low dive at the local swimming pool. It hurts a lot more if you belly flop off the high dive. It’s also why, for the same style of jump off either board, the splash from smacking the water after jumping off the high dive will always be bigger. You have more stored energy.

The same ideas apply for electric potential energy. When you use an electrical device, you are extracting energy from the electric charges. It’s like the splash when smacking the water after jumping off a diving
board. But before the energy can be extracted from the charges, they must first be given potential energy. And it’s done in the same way that energy is put into masses in a gravitational field. To get energy into charges, they are simply pushed in the opposite direction that they would normally move in an electric field.

**Voltage – The Most Misunderstood Concept in Electricity**

When I was about six-years-old, I touched the frayed end of a lamp cord and got a nasty shock. It felt like every muscle in my body tensed at once ... and that was probably the case. I learned a lesson in that one incident more than I could ever learn through any electrical safety awareness campaign. You don’t mess with the 120 volts of electrical potential that is poised for action at dozens of outlets throughout the typical household. However, ask me if I would be willing to get a 25,000-volt shock and my answer would be: “Sure, I’ve had many high voltage shocks, many much higher than 25,000 volts. But I have to decide how the shock will be administered and from what source it will come from.” Most people would not understand my great respect for and fear of the household 120 volts given my cavalier attitude toward much higher voltages. Don’t misunderstand; 25,000 volts can easily be fatal in some situations. It just doesn’t always have to be dangerous. I’m sure you’ve had plenty of shocks that were many thousands of volts.

The problem with most people’s understanding of voltage is that they equate high voltage with high energy. But the voltage of the electricity standing by to power up the electrical appliances in your home is not potential energy; it’s potential energy per charge:

\[ V = \frac{E}{q} \quad \text{and} \quad E = Vq \]

1 volt = 1 joule of energy per 1 Coulomb of charge

\[ \Rightarrow 1 \text{ volt} = \frac{1 \text{ Joule}}{\text{Coulomb}} \]

So it’s impossible to judge the danger of a shock by simply considering the voltage alone. 120 volts simply means that when a full Coulomb of charge has flowed into an electrical device, 120 joules of energy will have been delivered as well. If that deposit of one Coulomb of charge occurs over a tenth of a second, it hurts. But if it occurs over an hour, you wouldn’t notice it. The energy necessary to lift a half-cup of water one meter high is approximately equal to one Joule. Now the typical electrical outlet will allow perhaps 20 Coulombs of charge to flow out of it per second. That means that if it is a 120-volt outlet, the potential amount of energy released per second is:

\[ E = \left( 120 \text{ joules/coulomb} \right) \left( 20 \text{ coulombs} \right) = 2400\text{J} \]

That’s a lot of energy (enough to lift 75 gallons of water 1-meter high every second). Now consider the shock from the static discharge of a Van de Graff generator. These generators are capable of raising the electric potential of charge up to around 400,000 volts. However, it can only deliver charge at a rate of about 10 micro-Coulombs per second. So the energy released over a second from one of these generators is:

\[ E = \left( 4 \times 10^5 \text{ joules/coulomb} \right) \left( 10 \times 10^{-6} \text{ coulombs} \right) = 4\text{J} \]

(two cups of water lifted one meter high). By contrast, the energy from the high voltage Van de Graff generator shock is hundreds of times less than the shock from the household outlet.

I like to use the analogy of a waterfall when I explain voltage. The danger of being below a waterfall depends on two things: the height of the waterfall and...
the amount of water flowing over it. The height of the waterfall is like voltage. Double the height of a waterfall and you double the amount of potential energy in each of the individual drops at the top of the waterfall. But you haven’t necessarily doubled the danger by doubling the height. If less water falls from the taller waterfall, it could actually be less dangerous. Using this analogy, the voltage of the household outlet is like a short waterfall with a torrent of water flowing over it. Individual drops of water don’t have much energy, but when combined with many, many others, the total amount of energy is potentially very high. But the Van de Graaff generator is like a very tall waterfall with only a trickle falling over the edge. Lightning is high voltage, but always dangerous. Its 100,000,000 volts of electric potential with 30,000 to 40,000 Coulombs of charge flowing per second is like Niagara Falls with huge amounts of water flowing over the top.
CHAPTER 8:
ELECTRIC CIRCUITS

The first intentional human circuit was William Kemmler, when he was executed on August 6, 1890 (Figure 8.1). He had killed Tillie Ziegler in Buffalo, New York over a year earlier on March 29, 1889. Kemmler became the first person ever executed by electrocution – the first victim of New York’s newly designed electric chair. When a drunken man was accidentally electrocuted in a Buffalo electric generating plant on August 8, 1881, the idea for using electrocution was proposed as a more “humane” means of executing prisoners than by hanging them. Certainly people had been shocked before, but in order to provide a reliably lethal jolt, a constant source of electricity was required. A static discharge, no matter how powerful, could not be relied upon to absolutely kill the condemned (after all, even those struck by lightning sometimes live). No, a continuous source of electricity, a current of electricity constantly flowing within a circuit, was required.

The Simplest Electric Circuit

The electrophorous and the Van de Graff generator can both be used to store charge and, in the process, store energy. In a static discharge from either of these devices, the stored charge is given a path to the ground and the stored energy in that charge can be removed. Put more simply, if you touch a charged-up Van de Graff generator, you get shocked as the charge flows through you to the ground. But it’s a transient event. You only feel the energy (thankfully) for a moment, and then all the excess charge is gone. In order to deliver charge (and thus, energy) continuously, you need to use a circuit. A circuit, in its simplest form, contains three components: a source of electrical potential, a resistance load, and conductors to connect the source to the load. Figure 8.2 illustrates what the simplest circuit looks like. (For contrast, consider the much more complicated circuit shown in Figure 8.4.)

Figure 8.1: A drawing of William Kemmler (left), the first intentional human circuit, shows his death on August 6, 1890 in the first ever execution by electric chair. A photograph of an early electric chair is shown on the right.

Figure 8.2: The simplest circuit consists of a source of electric potential, a resistance load, and conductors to connect the two. In this circuit (created by Braden Hoyt, Class of 2008), the light bulb filament is the load. The conductors include not just the wire connected to the light bulb, but also the graphite in the pencils and the water in which the pencils are submerged.
In a circuit, the source provides electric potential (voltage) to the charges in the circuit. There are varieties of methods to do this. Each method converts some type of energy to electric potential. The battery converts the energy of a chemical reaction to electric potential. As long as there is un-reacted chemical in the dry cell battery, it will continue to provide the voltage at which it is rated. The solar cell on a calculator converts light energy to electric potential. Microphones convert mechanical energy to electric potential. The sound waves hitting a microphone apply a pressure to a membrane known as a piezoelectric device, causing a voltage difference to occur on each side of the device. The electromagnetic generators used by electrical power companies also convert mechanical energy to electric potential. As large coils of wire are forced to rotate in a magnetic field, a potential is created in the coil. Regardless of the method, the source takes electric charge with no potential and gives it potential.

The load in a circuit uses up the energy that has been given to the charge. The load can use up this energy three ways. The load can convert the electric energy to light (like a light bulb does). Another way to use the electric energy is by converting it to heat (like in a toaster or in the steel wool-battery circuit pictured in Figure 8.3). The final method for converting electric energy is into motion (like in the motor of a washing machine). Most loads convert electric energy to more than one of these. The electric incandescent light bulb, for example, is a much better heater than an illuminator. About 80% of its energy is in the form of heat. And a quick look at the wires in a toaster shows that they glow as well as heat.

The conductors in an electric circuit are usually obvious. The power cord coming out of many electric devices provides a path for the electric energy to flow from the outlet to the device. The electric power lines strung along power poles or buried underground provide a path for the electric energy to flow from the generating facility to homes and businesses.

**Figure 8.3:** The steel wool and 9-volt battery in this photograph (taken by Daanika Gordon, class of 2005) constitute a very simple circuit. The battery provides a source of electric potential and the steel wool acts as both the conductor and the resistance load. This load converts the energy from the source to heat (enough to catch the steel wool on fire!)

**Figure 8.4:** This circuit board, taken from a household central heating furnace, illustrates a more complex circuit than the simplest circuit discussed above.

The load in a circuit uses up the energy that has been given to the charge. The load can use up this energy three ways. The load can convert the electric energy to light (like a light bulb does). Another way to use the electric energy is by converting it to heat (like in a toaster or in the steel wool-battery circuit pictured in Figure 8.3). The final method for converting electric energy is into motion (like in the motor of a washing machine). Most loads convert electric energy to more than one of these. The electric incandescent light bulb, for example, is a much better heater than an illuminator. About 80% of its energy is in the form of heat. And a quick look at the wires in a toaster shows that they glow as well as heat.

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**ELECTRICAL RESISTANCE**

When charge moves through a circuit, it hardly ever does it effortlessly. There is almost always some resistance to the flow of charge. The only exception is in materials known as superconductors. Some materials have a very large resistance to the flow of electricity. These materials are referred to as insulators. Rubber, plastic, and glass are all highly electrically resistant. Other materials (conductors) have very little electrical resistance. These are materials like copper, silver, and other metals. It’s a bit more complicated than that though because two wires of copper can have very different resistance …
if their geometry is different. The resistance of a wire not only depends on the type of substance it is made of, but also the length and cross-sectional area of the wire. The longer the wire is, the higher the resistance it will have and the larger its diameter, the less its resistance will be. It might help to think about blowing air through different straws. Imagine a very thin coffee stirrer and a large diameter soda straw. It is much easier to push air through the large straw and the same is true when trying to push charge through a wire (Figure 8.5). Now imagine that you had two straws that were the same diameter, but one was one centimeter long and the other was one meter long. It would be almost effortless to blow through the short straw, compared to the long one.

To find the resistance, \( R \), of a particular substance you need to know its length, \( L \), its cross-sectional area, \( A \), and its resistivity, \( \rho \) (this distinguishes one substance from another):

\[
R = \rho \frac{L}{A}
\]

Table 8.1 gives the resistivity of some conductors and insulators.

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity, ( \rho ) (( \Omega \cdot \text{m} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>1.59 x 10^{-8}</td>
</tr>
<tr>
<td>Copper</td>
<td>1.68 x 10^{-8}</td>
</tr>
<tr>
<td>Tungsten</td>
<td>5.6 x 10^{-8}</td>
</tr>
<tr>
<td>Nichrome</td>
<td>1.0 x 10^{-6}</td>
</tr>
<tr>
<td>Graphite</td>
<td>1 x 10^{-4}</td>
</tr>
<tr>
<td>Silicon</td>
<td>0.1 - 60</td>
</tr>
<tr>
<td>Dry Air</td>
<td>3 x 10^{6}</td>
</tr>
<tr>
<td>Glass</td>
<td>10^{9} - 10^{12}</td>
</tr>
<tr>
<td>Rubber</td>
<td>10^{13} - 10^{15}</td>
</tr>
</tbody>
</table>

Table 8.1: Resistivities of various materials

One thing to notice about Table 8.1 is the HUGE range in the electrical resistivity between different substances. Those with the very small resistivities are the conductors and those with the very high resistivities belong to the insulators. However, in one sense, these are all conductors. It’s just that the ones with higher resistivity require more voltage to drive a current through the substance. (Figure 8.6 shows a 100,000-volt stun gun causing electricity to flow across a few centimeters of dry air.)

Also notice that silicon doesn’t fit easily into either conductor or insulator. So, silicon is known as a semiconductor.

The Greek omega (\( \Omega \)) is the symbol for the unit of electrical resistance, the Ohm. The ohm unit is in honor of the German physicist, Georg Ohm. He had a very mercurial life. He was unhappy through most of it. Even after demonstrating the relationship, or law, which would someday be named after him, most of his colleagues were unimpressed. It wasn’t until just before his death that he was finally given his long sought after professorship.

Ohm was the one who first understood and explained the connection between the voltage applied to a circuit, the circuit’s resistance, and the current that would subsequently flow. Eventually, this explanation became known as Ohm’s Law.

Figure 8.5: This photograph (taken by Sophia Carmen, Class of 2009) shows a “water circuit” made with straws of different diameters. This is analogous to the electrical circuit and its higher current in wires with greater radius.

Figure 8.6: The high resistivity of air would classify it as an insulator. However, with a high enough voltage applied, insulators can be made to conduct electricity. Here the 100,000 volts produced by this stun gun causes electricity to flow between its electrodes. (Photo by Ryan Villanueva, Class of 2008.)
Example
How much electrical resistance does the graphite in a 20-cm long pencil have if the graphite has a diameter of 4.0-mm?

Solution:

- Identify all givens (explicit and implicit) and label with the proper symbol.
  - The 0.20-m is the length, \( L \)
  - the 4.0-mm can be used to calculate the cross-sectional area, \( A \).

\[
A = \pi r^2 = \pi \left(\frac{0.0040\text{ m}}{2}\right)^2 = 1.26 \times 10^{-5}\text{ m}^2
\]

- The substance is graphite. Therefore, \( 1 \times 10^{-4}\text{ Ω} \cdot \text{m} \) is the resistivity, \( \rho \)

Given:

\[
L = 0.20\text{ m} \quad A = 1.26 \times 10^{-5}\text{ m}^2 \\
\rho = 1 \times 10^{-4}\text{ Ω} \cdot \text{m}
\]

- Determine what you’re trying to find.
  - The question explicitly asks for electrical resistance.

Find: \( R \)

- Do the calculations.

\[
R = \rho \frac{L}{A} = \left(1 \times 10^{-4}\text{ Ω} \cdot \text{m}\right) \left(\frac{0.20\text{ m}}{1.26 \times 10^{-5}\text{ m}^2}\right) = 1.59\text{ Ω}
\]

Ohm’s Law

If you want to light up a dark room, you turn on the light switch. And, like any circuit, when the switch is turned on, a steady current of charge, \( I \), will begin to flow and deposit its energy in the load of the circuit. This current is the rate of flow of charge past any point in the circuit:

\[
\text{Current} = \frac{\text{Charge}}{\text{time}} \Rightarrow I = \frac{q}{t}
\]

where \( q \) = charge, measured in Coulombs
\( t \) = time, measured in seconds

1 Ampere (A) = \( \frac{1 \text{ Coulomb}}{\text{second}} \)

The amount of current that flows depends on two things: the potential difference, \( V \), of the circuit and the resistance, \( R \), of the circuit. Ohm showed that the current was proportional to the potential difference and inversely proportional to the resistance in the circuit:

\[ I \propto V \text{ and } I \propto \frac{1}{R} \]

These two ideas are combined together to convey Ohm’s Law:

\[ I = \frac{V}{R} \]

also expressed as

\[ V = IR \quad \text{ or } \quad R = \frac{V}{I} \]

Don’t go forward until you understand this conceptually. You can’t see charge, so you have to linger with ideas about electricity a bit longer. Try comparing the electrical circuit to a water circuit. In a water circuit, the analogy of the potential difference is the pressure of the water. The resistance has to do with the thickness (and length) of the hose, the switch is like the water spigot, and the current is the rate of water flow in the hose. The water circuit clearly follows Ohm’s Law, because the flow of water increases when the pressure increases. Also, the connection between current and resistance is clear because if the hose gets a kink and the resistance increases, it causes a smaller flow of water through the hose.

Electric Shocks

The discussion so far has been very technical, but when the average person thinks about electricity, it is generally not about voltages and currents, but about electrical devices and then perhaps about electric shocks. Most people have experienced an electric shock. If you have, it’s because you became the load in an electrical circuit and the energy in that electricity was deposited in … you, and the sudden pain of that energy “shocked” you. Now you’ve probably had many shocks and never even realized it. In order to feel a shock (but not really feel pain), the current needs to be between 10 \( \mu \text{A} \) and 100 \( \mu \text{A} \). But once the current reaches 1 mA, you definitely begin
to feel discomfort and pain. If the current is higher still, something curious happens. At about 10 mA, the release current is reached. You may be aware that muscles contract due to electrical signals. If you get a shock that produces a current as high as the release current, it overrides the body’s ability to contract muscles. The release current causes all muscles to contract. This is a big problem in the case where someone has grabbed hold of a source of electricity. There are muscles that cause the hand to squeeze shut and others to open up, but both are being stimulated at the same time. The squeezing muscles are stronger than those muscles that open the hand, so the person grabbing a source of electricity that causes the release current to be met or exceeded … can’t let go! Of course, it’s possible that the shock can be much greater than the release current. Death will occur at currents of approximately 100 mA. (For perspective, the current produced in the electric chair can reach 12 A).

Although 100 mA (0.1 A) can be lethal, it is small when considering those common in many ordinary electrical devices (hair dryers can draw more than 12 A). Fuses and circuit breakers are tripped when currents on the order of 20 A or 30 A are reached, so it’s unreasonable to expect that a fuse or circuit breaker would be tripped by the small current of a lethal shock. However, ground fault interrupts (those trip switches that are built into individual outlets) are sensitive down to 5 mA. The ground fault interrupt looks at the disparity between the current entering and exiting a circuit. So if the device that you get a shock from delivers a current in excess of 5 mA, the ground fault interrupt trips the circuit and stops the current from flowing.

**POWER AND ENERGY IN CIRCUITS**

Every time I’m in Costco in the wintertime with my wife, I try to avoid the area where the electric space heaters (the ones with the parabolic mirrors that focus the heat) are located. If we end up walking by them, there’s always an appeal to impulse-buy one. I’ve got to admit, it feels really good to stand in front of one when it’s freezing outside. There’s plenty of electrical power being generated to create all that heat. I think the easiest way to understand electrical power is to look at the units for voltage and current. But first it’s important to remember that energy is measured in Joules. Power is related to energy – it is the rate of energy flow. Therefore, power would have the units of Joules/second. We'll actually define a new unit for power, the Watt:

\[ 1 \text{ Watt (W)} = \frac{1 \text{ Joule}}{\text{second}} \]

Now let’s look at the units for voltage and current. Remember:

\[ \text{volt}(v) = \frac{\text{Joule}}{\text{Coulomb}} \text{ and current}(I) = \frac{\text{Coulomb}}{\text{second}} \]

That means: \((V)(I)\) gives the units of:

\[ \left( \frac{\text{Joules}}{\text{Coulomb}} \right) \left( \frac{\text{Coulomb}}{\text{second}} \right) = \frac{\text{Joules}}{\text{second}} \]

But, Joules/second is the unit for power. So calculating electrical power is easy:

\[ P = VI \]

To calculate the energy produced in a circuit (like the total amount of heat produced by a space heater), you simply have to multiply the power produced by the time that the circuit is in operation:

\[ E = Pt \text{ or } E = VI t \]

What if you wanted to build one of those space heaters so that it would put out 1500 W of power? If it were to be plugged into a 120-V outlet, you could calculate the necessary current and then with the current and voltage, you could calculate the required resistance. But it’s easier to make one calculation by combining Ohm’s Law with the equations for power and energy. You know that Ohm’s Law can be expressed as either \( I = \frac{V}{R} \) or \( V = IR \). Both of these can be substituted into the equations for power and energy to get the following equations:

\[ P = VI \Rightarrow P = V \left( \frac{V}{R} \right) \Rightarrow P = \frac{V^2}{R} \]

\[ P = VI \Rightarrow P = (IR)I \Rightarrow P = I^2R \]
And since \( E = Pt \):

\[
E = \frac{V^2}{R} t \quad \text{and} \quad E = I^2Rt
\]

We started this section with the simplest circuit. It makes sense to finish it with the most complicated circuit – The Grid. The Grid is the United States electricity system. In 2003, the US National Academy of Engineering named The Grid the greatest engineering achievement of the 20th Century. It began in 1881 by Thomas Edison and his team in the Wall Street area of New York City. (There are still vestiges of this early system providing electrical power to a small group of 2,000 customers.) Today, The Grid consists of more than 200,000 miles of high voltage (\( \geq 230kV \)) lines bringing service to more than 300,000,000 customers.

**Example**

In 3.0 minutes an electric coffee pot delivers 48,000 J of energy to the water inside it. The coffee pot is connected to a standard 120-volt source. What is the resistance of the coffee pot?

**Solution:**

- Identify all givens (explicit and implicit) and label with the proper symbol.
  - The 3.0 minutes is time, \( t \), but must be converted to seconds.
  - The 48,000 J is energy, \( E \).
  - The 120-volts is potential difference, \( V \).
- Determine what you’re trying to find.
  - The question explicitly asks for electrical resistance.

**Given:**

- \( t = 180 \text{ seconds} \)
- \( E = 48,000 \text{ J} \)
- \( V = 120 \text{ V} \)

**Find:** \( R \)

- Do the calculations.

\[
E = \frac{V^2}{R} t \quad \Rightarrow \quad R = \frac{V^2}{E} t = \frac{(120\text{V})^2}{48,000\text{J}} \cdot 180\text{s} = 54 \Omega
\]

**Biography – Georg Ohm**

(Written by Alex Altman, class of 2005)

Georg Ohm (1789-1854)

If you’ve felt that absolutely no one understands you, you’re not alone. German physicist Georg Ohm lived most of his scientific life in frustration, struggling to gain acceptance for his scientific theories.

What is known as “Ohm’s Law,” the relationship between current, resistance, and voltage, \( I = \frac{V}{R} \) was first described in his most famous work, “Die Galvanische Kette, Mathematisch Bearbeitet” (1827) (The Galvanic Circuit, Investigated Mathematically) in which he gave his complete theory of electricity. His work was not received well by the scientific community. The accepted practice at the time was to approach physics from a non-mathematical perspective. Ohm did the exact opposite. The book begins with the mathematical background necessary for understanding the rest of the book. Most of Germany’s top scientists couldn’t get past this part. Ohm’s disjointed mathematical proofs made him an object of ridicule. His feelings were hurt and he resigned his position as professor at Cologne University.

In 1842, after Ohm spent many years lecturing around the country, the Royal Society in London finally recognized him for his discoveries in electricity. In 1849, just five years before his death, Ohm realized his lifelong dream of becoming the head of physics at Munich University.
THE KILOWATT-HOUR

The box above is from a portion of the author’s PG&E bill. Note that the electric charges are not in the conventional energy unit of Joules, but in Kwh. This is the abbreviation for kilowatt-hour. The amount of electrical energy used by most devices is large enough that the Joule would be an awkward unit to use (something like an automobile speed limit being specified in inches per day). To understand the energy of a kilowatt-hour, consider a bright 100-watt light bulb. Lighting ten of them at once would use 1000 watts of power – one kilowatt. The amount of energy necessary to light these ten light bulbs for one hour would be one kilowatt-hour. The portion of the electric bill above shows that the cost of this amount of energy (in December 2008) is 11.55¢. This amounts to only a little over 1¢ per 100-watt light bulb per hour, which makes electrical energy sound inexpensive. It sounds even more inexpensive if you compare kilowatt-hours and Joules. One Joule of energy is what is required to apply one Newton of force through a distance of one meter, and since a Newton of force is about one-quarter of a pound, you can think of one Joule as the amount of energy that would be needed to lift a stick of butter waist high. Now let’s compare Joules and kilowatt-hours. Remember, 1 Watt = 1 Joule/second:

\[
(1kW \cdot hr) \left( \frac{1000W}{1kW} \right) \left( \frac{1J/s}{1W} \right) \left( \frac{3600s}{1hr} \right) = 3,600,000J
\]

Your electrical energy calculations will always conventionally use Joules, but practically the electrical energy purchase is in kilowatt-hours. Use the conversion of 1 kW-hr = 3,600,000 J when necessary.

Finally, to appreciate the extraordinarily inexpensive cost of electrical energy, consider the amount of mechanical energy that is equivalent to one kilowatt-hour. If the cost of one kilowatt-hour is 11.55¢, then for a bit more than a dime you can purchase enough electrical energy to do the equivalent of lifting 3,600,000 sticks of butter waist high. That’s cheap!

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Symbol</th>
<th>Unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge</td>
<td>Q or q</td>
<td>Coulomb</td>
<td>C</td>
</tr>
<tr>
<td>Current</td>
<td>I</td>
<td>Ampere or “Amp”</td>
<td>A</td>
</tr>
<tr>
<td>Potential Difference</td>
<td>V</td>
<td>Volt</td>
<td>V</td>
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<tr>
<td>Time</td>
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</table>

Table 8.2: Current electricity measurements and symbols
CIRCUITS WITH MORE THAN ONE LOAD

Occasionally you still find those strings of lights used for decorating Christmas trees that can cause major frustration. If any one of the many bulbs in the string burns out, they all go out. The only way you can find which is the bad one is by taking a known good bulb and using it in place of each of the bulbs in the string until the string lights up again. The bulbs in those strings of lights are wired in a series circuit, which means that there is only one path for the electricity to follow (Figure 8.7, 8.8). So if the filament in one of the bulbs burns out, current can no longer flow. It may seem like a poor way to wire a circuit, but it is the way that all fuses and circuit breakers are inserted into the wiring of houses and other buildings. If the fuse burns out it shuts down the current in the entire circuit so that a potential fire doesn’t result. Circuits can also be wired as parallel circuits, which means there are at least two paths for current to take (Figure 8.7, 8.9). This way if one part of the circuit fails, current still flows through the other parts of the circuit. This is important for circuits like the one in your kitchen. Since that circuit is wired in parallel, when you turn off the lights in the kitchen the refrigerator continues to operate. There is one final circuit, the combination circuit, which combines both series and parallel portions in the same circuit. We’ll look at that one in the next lab.

If you look at Figures 8.8 and 8.9, it’s probably not entirely clear that the various aspects of the circuits like the voltages, currents, and resistances are completely different. For most people, the most surprising (and counterintuitive) idea is that the total resistance in a parallel circuit is smaller than the smallest resistance in the circuit. This section is meant to help you understand and make sense of the various aspects of series and parallel circuits. Specifically, we’ll look at the current, voltage drops, and resistance of the total circuit as well as that of specific parts of the circuit.
SERIES CIRCUITS

Series circuits are very simple – there is only one path for the current to flow through (see Figure 8.10). So let’s start by thinking about this current. If current is flowing in the circuit, and there is only one path, then the rate of the flow must be the same no matter where you look. It would be like measuring the flow of water in a garden hose. No matter what point in the hose you checked, you would always find the same volume of water passing that point per second. This means that the total current flowing from the source is the same as that flowing through any of the resistors:

$$I_T = I_1 = I_2 = I_3$$

Now let’s consider how the individual resistors in a series circuit compare to the total resistance of the circuit. First think about a very simple series circuit with only one resistor (say a single light bulb). If you took the light bulb filament (which is the resistor), cut it in half, and separated it with a conductor, you would have a series circuit with two identical resistors. The total resistance of the circuit would be the same as before the filament was cut. This logically leads to the total resistance in a series circuit being equal to the sum of the individual resistors:

$$R_T = R_1 + R_2 + R_3$$

Finally, let’s look at how the total voltage of the source compares to the individual voltage losses (drops) in the series circuit. Remember, the voltage provided by the circuit is simply the energy supplied for a particular amount of charge – one volt guarantees one Joule of energy for every Coulomb of charge. So if there were a wire connected between both poles of a battery, the charge pouring out of the negative pole of the battery would give up all its energy heating up the wire on its journey to the positive side of the battery. Now if the wire were replaced by resistors, the voltage would be lost at the resistors instead. I hope that it makes sense that the energy in the charge would be lost proportionately (twice as much energy for twice as much resistance) as the charge flows through them, moving from the negative pole to the positive pole:

$$V_T = V_1 + V_2 + V_3$$
**Parallel Circuits**

Parallel circuits are more complex (see Figure 8.11). You should think of them as multiple series circuits connected to the same source. The series circuit requires that every charge flow through every resistor, but the parallel circuit allows each charge to flow through only one resistor. That means that when a charge chooses a path and flows through a particular resistor, it will (and must) lose all its energy at that resistor. That is, the voltage drop at any resistor in a parallel circuit is equal to the total voltage of the source:

\[ V_T = V_1 = V_2 = V_3 \]

Now, since the charges flowing in a parallel circuit are really navigating one of multiple series circuits, there are actually multiple currents flowing at once, each through its own isolated circuit. The total current flowing from the source must be enough to provide for all these isolated circuits. That is, the total current in a parallel circuit is the sum of the currents flowing in all the individual branches of the circuit:

\[ I_T = I_1 + I_2 + I_3 \]

Finally, let’s look at the total resistance in a parallel circuit. First we’ll use the result about current in a parallel circuit:

\[ I_T = I_1 + I_2 + I_3 \]

Now let’s rewrite the same equation, but this time substituting in Ohm’s Law \( I = \frac{V}{R} \):

\[ \frac{V_T}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \]

But, remember that \( V_T = V_1 = V_2 = V_3 \), so we could drop all the subscripts and say:

\[ \frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \]

If we divide both sides of the equation by \( V \), the equation that relates total resistance in a parallel circuit to the individual resistances becomes:

\[ \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]

This means that for a parallel circuit with three resistors that have values of 100 Ω, 200 Ω, and 300 Ω, the total resistance of the circuit is less than 100 Ω:

\[ \frac{1}{R_T} = \frac{1}{100\Omega} + \frac{1}{200\Omega} + \frac{1}{300\Omega} \]

\[ \Rightarrow \frac{1}{R_T} = \frac{6}{600\Omega} + \frac{3}{600\Omega} + \frac{2}{600\Omega} = \frac{11}{600\Omega} \]

\[ \Rightarrow R_T = \frac{600\Omega}{11} = 55\Omega \]
Most people feel that it is counterintuitive for the total resistance in a parallel circuit to be smaller than the smallest resistance in that circuit. Try thinking of a lake full of water with a dam at one end. The dam is like an insulator, preventing the flow of water. If a hole were drilled through the dam, it would be a path for water to flow, but it would resist the flow more than if the dam weren’t there at all. So, the hole in the dam is like a resistor in an electrical circuit (both allowing a flow of current and restricting it at the same time). Drilling another hole in the dam would add another resistor, but it would also allow more water to flow than when there was only one hole in the dam. So the second hole in the dam would actually reduce the overall resistance to current flow—the total resistance would be lower than if there were fewer resistors. The key here is to understand that at the same time that resistors truly resist the flow of electricity, they are ultimately paths through which charge can flow.

Figure 8.12: In the series circuit on the left, charges must flow through all three bulbs. However, in the parallel circuit on the right charges have three possible paths and will only flow through the one light bulb in the path they choose. This means that the charges in the series circuit experience three times the resistance and therefore, glow less brightly. Also, the charges in the series circuit have only one-third the voltage drop at each light bulb compared to the parallel circuit. In the series circuit, three times the resistance means one-third the current. Combining this with the one-third voltage drop means that the power in a light bulb of the series circuit produces only one-ninth the power of a light bulb in the parallel circuit. (Photo by Carrie Coats, Class of 2008.)
**Combination Circuits**

Combination circuits have elements of both series circuits and parallel circuits. Figure 8.13 compares these three circuits side-by-side. Let's review the essential elements of the series and parallel circuits a bit. The series circuit consists of only one path. This means that the current is identical everywhere and since individual charges must pass through each of the resistors, the voltage drops are portioned out proportional to the resistance of the resistor. The parallel circuit, by contrast, has a separate path for each resistor. This means that the current is not identical everywhere. The total current flowing from the source gets portioned out to the individual resistors according to the size of the resistor (smaller resistors get larger currents). Now, since an individual charge in the parallel circuit can only pass through one of the resistors, it drops all of its voltage there (the full voltage of the source).

The current and voltage drop rules for series and parallel circuits are so different it might seem very complicated to have features of both circuits in the same combination circuit. It’s actually not as tricky as it might seem. Your strategy should always be to follow this three-step process:

1. Replace combinations of resistors with single resistors so as to gradually convert the combination circuit to a series circuit.
2. Find the total resistance of the equivalent circuit.
3. Find the total resistance of the equivalent circuit.

After completing the process above, you can apply Ohm’s Law and the other circuit equations as you did with simple series and parallel circuits.